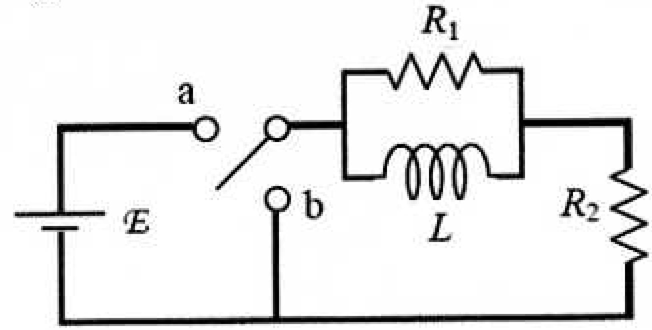
Inductors.

1. The diagram shows a classic *RL* circuit, containing both resistors and inductors. The switch is shown initially connected to neither terminal, and is then thrown to position "a" at time $t = 0$.



$\mathcal{E} = 10$ V
$L = 15$ mH
$R_1 = 4\ \Omega$
$R_2 = 6\ \Omega$

a. At $t = 0^+$, just after the switch is thrown to position a, what are the currents I_1 and I_2 across the two resistors?

No current through L

$$I_1 = I_2 = \frac{\mathcal{E}}{R_1 + R_2} = \boxed{1.0\text{A}}$$

b. After a very long time, what is the instantaneous power dissipated in the circuit?

No current through R_1 (L is a short)

$$I = \frac{\mathcal{E}}{R_2} \qquad P = I^2 R_2 = \frac{\mathcal{E}^2}{R_2} = \frac{(10\text{V})^2}{6\Omega} = \boxed{16.7\text{W}}$$

c. What is the final energy stored in the inductor, after a very long time?

$$U_f = \frac{1}{2} L I^2 = \frac{1}{2}(15 \times 10^{-3}\text{ H})(1.67\text{A})^2 = \boxed{0.021\text{J}}$$

$$I = \frac{\mathcal{E}}{R_2} = \frac{10\text{V}}{6\ \Omega} = 1.67\text{A}$$

d. Sketch the behavior of the energy stored in the inductor as a function of time.

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Now, after a long time, the clock is reset to 0 and the switch is thrown to position b.

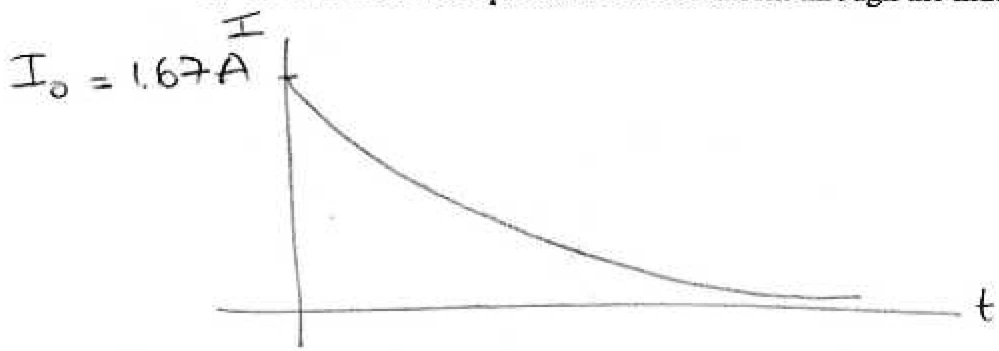
e. What is the time constant τ describing the change in current through the inductor?

Redraw circuit:

$$R_{eff} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = 2.4\,\Omega$$

$$\tau = \frac{L}{R} = \frac{15\,\text{mH}}{2.4\,\Omega} = \boxed{6.3\,\text{ms}}$$

f. Sketch the time dependence of the current through the inductor.



g. What is the energy stored in the inductor 8.0 ms after the switch goes to position b?

$$I = I_0 e^{-t/\tau}$$

$$U = \frac{1}{2} L I^2 = \frac{1}{2} L I_0^2 e^{-\frac{2t}{\tau}}$$

$$U(8\,\text{ms}) = \frac{1}{2}(15\,\text{mH})(1.67\,\text{A})^2 e^{-\frac{16\,\text{ms}}{6.3\,\text{ms}}}$$

$$= \boxed{1.61\,\text{mJ}}$$

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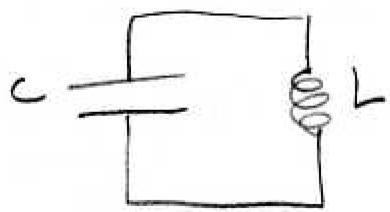
2. Your latest invention is a car alarm that produces sound at a particularly annoying frequency of 3500 Hz. To do this, the car alarm circuitry must produce an alternating electric current of the same frequency. That's why your design includes an inductor and a capacitor in series. The maximum voltage across the capacitor is to be 12.0 V (the same voltage as the car battery). To produce a sufficiently loud sound, the capacitor must store 0.0160 J of energy. What values of capacitance and inductance should you choose for your car alarm circuit?



$f = 3500\,Hz$

$V_{C\,max} = 12.0V$

$U_{max} = 0.0160J$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$V_{C\,max} = \frac{Q_{max}}{C} \qquad Q_{max} = V_{C\,max}\,C$$

$$U_{max} = \frac{1}{2}\frac{Q_{max}^2}{C} = \frac{1}{2}V_{C\,max}^2\,C$$

$$C = \frac{2U_{max}}{V_{C\,max}^2} = \frac{2(0.0160J)}{(12.0V)^2} = \boxed{2.2\times10^{-4}\,F}$$

$$L = \frac{1}{\omega^2 C} = \frac{1}{(2\pi f)^2 C} = \frac{1}{(2\pi\cdot3500)^2(2.2\times10^{-4}F)}$$

$$= \boxed{9.3\times10^{-6}\,H}$$