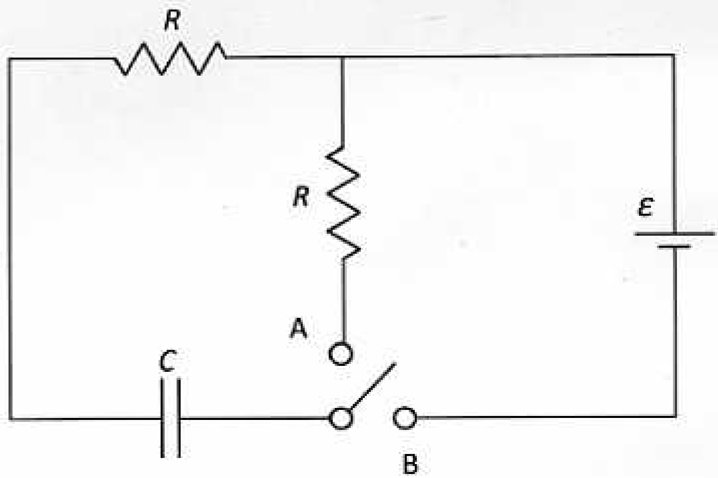
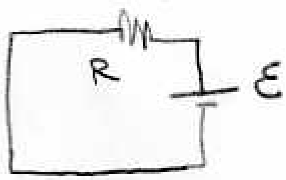
2. In the RC circuit below, the switch has been open for a long time. At $t = 0$, the switch is moved to position B.



$\varepsilon = 30$ V
$R = 15\ \Omega$
$C = 5.0\ \mu$F

a. What is the current through the battery right after the switch is closed?

Effective circuit at $t = 0^+$



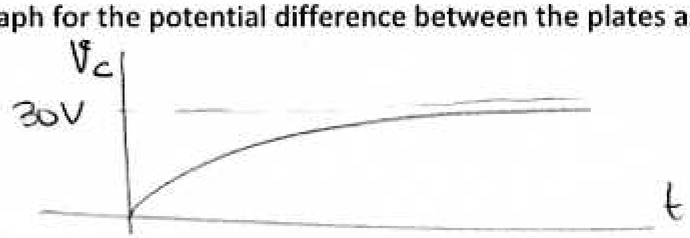
$I = \frac{30V}{15\Omega} = \boxed{2A}$

b. What is the potential difference between the plates of the capacitor at $t = 100\ \mu$s?

$V(t) = V_\infty\left(1 - e^{-t/\tau}\right) = \varepsilon\left(1 - e^{-t/\tau}\right)$

$(\tau = RC = 75\mu s)$ $= 30V\left(1 - e^{-\frac{100}{75}}\right) = \boxed{22V}$

c. Sketch a graph for the potential difference between the plates as a function of time.



d. How much charge is stored in the capacitor a very long time after the switch is moved?

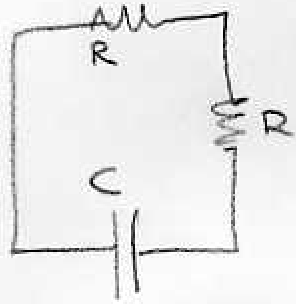
$Q(\infty) = C\varepsilon = \boxed{150\mu C}$

(Turn over)

After the capacitor is fully charged, the switch is moved to A.

e. What is the current through point A right after the switch is moved?

Effective circuit



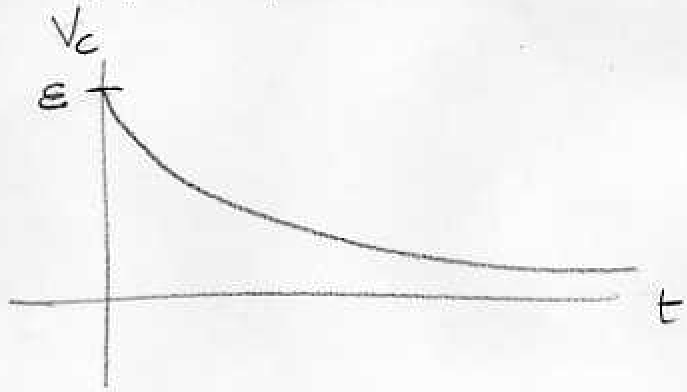
$$I = \frac{V_c}{2R} = \frac{\varepsilon}{2R} = \frac{30V}{30\Omega} = \boxed{1A}$$

f. What is the time constant for the discharge of the capacitor through this circuit?

$$\tau = 2RC = \boxed{150\,\mu s}$$

(Discharge will take twice as long as charge)

g. Sketch a graph for the potential difference between the plates as a function of time.

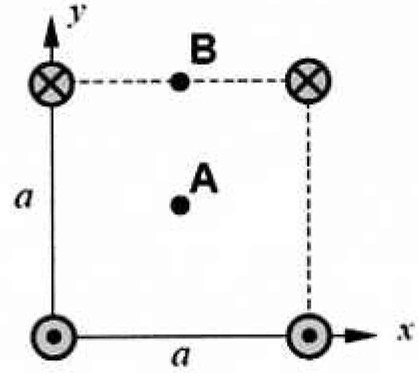


(Turn over)

(Magnetic forces)

2. The magnetic field for a infinite straight wire was presented in lecture:

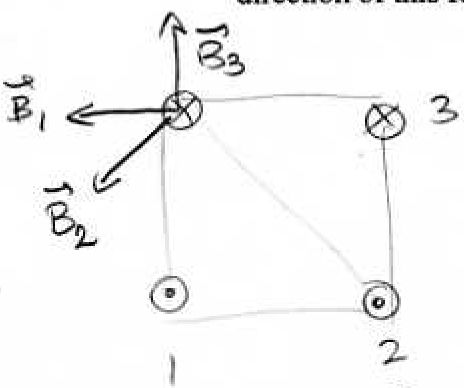
$$B = \frac{\mu_0 I}{2\pi r}$$



$a = 15$ cm
$I = 2.5$ A

Four long parallel wires are located at the corners of a square of side a. Each wire carries a current I. The top two currents are directed into the page, and the bottom two out of the page.

a. Calculate the force per unit length exerted on the wire in the top right corner by the field of the other three. Remember to give both the magnitude and direction of this force.



$$B_1 = B_3 = \frac{\mu_0 I}{2\pi a} \qquad B_2 = \frac{\mu_0 I}{2\pi a\sqrt{2}}$$

$$\vec{B}_{all} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3$$

$$\vec{B}_{all} = \frac{\mu_0 I}{2\pi a}(-\hat{i}) + \frac{\mu_0 I}{2\pi a\sqrt{2}}\left(\frac{-\hat{i}-\hat{j}}{\sqrt{2}}\right) + \frac{\mu_0 I}{2\pi a}(\hat{j})$$

$$= \frac{\mu_0 I}{2\pi a}\left(-\frac{3}{2}\hat{i} + \frac{1}{2}\hat{j}\right)$$

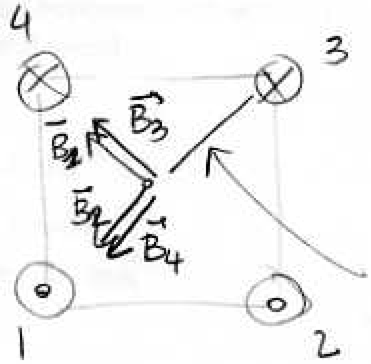
$$\vec{F} = I\vec{L}\times\vec{B} = I\frac{\mu_0 I}{2\pi a}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -L \\ -\frac{3}{2} & \frac{1}{2} & 0 \end{vmatrix}$$

$$= \frac{\mu_0 I^2}{4\pi a}\left(3L\hat{j} + L\hat{i}\right) = \frac{\mu_0 I^2}{4\pi a}L\left(\hat{i} + 3\hat{j}\right)$$

$$\boxed{\frac{\vec{F}}{L} = \frac{\mu_0 I^2}{4\pi a}\left(\hat{i} + 3\hat{j}\right)}$$

(Magnetic forces)

b. **Calculate the magnetic field vector B at point A (center of the square) and at point B.**

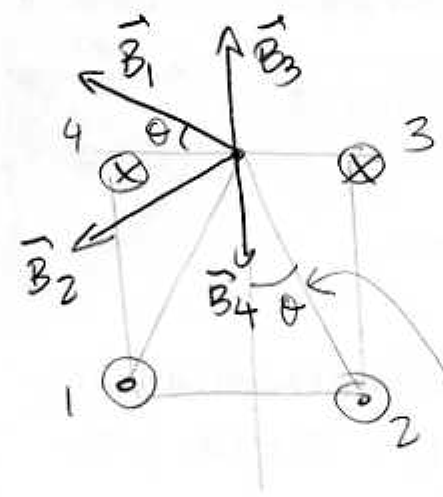


$$B_1 = B_2 = B_3 = B_4 = \frac{\mu_0 I}{\pi a\sqrt{2}}$$

$$r = \frac{a}{2}\sqrt{2}$$

Symmetry : only horizontal components matter :

$$\vec{B}_A = 4 \times \frac{\mu_0 I}{\pi a\sqrt{2}}\,\frac{\sqrt{2}}{2}\,(-\hat{\imath}) = \boxed{-\frac{2\mu_0 I}{\pi a}\,\hat{\imath}}$$



$$\vec{B}_3 + \vec{B}_4 = 0$$

$$B_1 = B_2 = \frac{2\mu_0 I}{2\pi a\sqrt{5}} = \frac{\mu_0 I}{\pi a\sqrt{5}}$$

$$r = \sqrt{\left(\frac{a}{2}\right)^2 + a^2} = \frac{a\sqrt{5}}{2}$$

$$\vec{B}_B = 2 \times \frac{\mu_0 I}{\pi a\sqrt{5}}\cos\theta\,(-\hat{\imath}) = 2\frac{\mu_0 I}{\pi a\sqrt{5}}\cdot\frac{2}{\sqrt{5}}\,(-\hat{\imath})$$

$$\cos\theta = \frac{a}{\frac{a\sqrt{3}}{2}} = \frac{2}{\sqrt{5}}$$

$$= \boxed{-\frac{4\mu_0 I}{5\pi a}\,\hat{\imath}}$$