

# Capacitance

1. Consider a parallel plate capacitor made of two large metal plates with area  $A$  separated a distance  $d$ .

- a. The system is connected to a battery that provides a constant potential difference  $V_0$  and allowed to charge. After that, the plates are disconnected from the battery. When the system is disconnected, the plates are pushed closer together to a distance  $d' = d/2$ . Explain how this changes (or does not change):

- The electric field between the plates

*If the system is isolated, the charge  $Q$  is constant. Therefore, the charge density  $\sigma = Q/A$  on the plates is constant, and so the electric field  $E = \frac{\sigma}{\epsilon_0}$  is the same.*

- The potential difference between the plates

*Since  $\Delta V = Ed$ , now the potential difference is half of what it was before.*

- The charge in each plate

*The charge remains the same (that's what we started from!)*

- The capacitance

*Capacitance doubles. To prove this, you can either use the formula  $C = \epsilon_0 \frac{A}{d}$  or argue that  $Q$  remains the same and  $V$  is halved, and  $C = Q/V$ .*

- The energy stored in the capacitor

*$U = \frac{Q^2}{2C}$ , so it is half of the initial value.*

- b. Suppose instead that the plates are pushed closer together to a distance  $d' = d/2$  while the system is still connected to the battery. Explain how this changes (or does not change):

- The electric field between the plates

*Now the potential difference is what remains constant. Since  $E = V/d$ , the electric field must be doubled.*

- The potential difference between the plates

*Constant (since it's fixed by the battery)*

- The charge in each plate

*The electric field doubled, and  $E = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$ , so  $Q$  must be doubled too.*

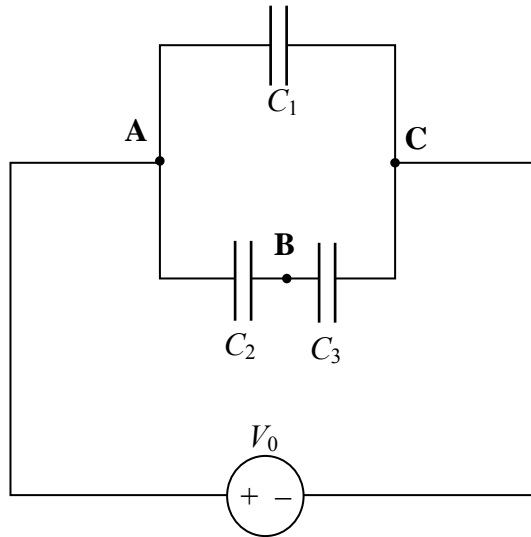
- The capacitance

*Capacitance doubles. To prove this, you can either use the formula  $C = \epsilon_0 \frac{A}{d}$  or argue that  $V$  remains the same and  $Q$  is doubled, and  $C=Q/V$ .*

- The energy stored in the capacitor. Where does the additional energy come from?

*$U = \frac{1}{2}CV^2$ , so it is twice the initial value.*

2. Three ideal capacitors  $C_1 = 1.0 \mu\text{F}$ ,  $C_2 = 2.0 \mu\text{F}$  and  $C_3 = 3.0 \mu\text{F}$  are connected as shown in the figure below to a source that provides a potential difference  $V_0 = 30 \text{ V}$ .



a. Compare the electric potential at points A and C (Choose one):

$$V_A > V_C$$

b. Indicate in the figure which plates have a positive/negative charge.

*All left-hand plates are positive. All right-hand plates are negative.*

c. Answer true or false to the following relations between charges on each capacitor:

$$Q_1 = Q_2 \quad \text{False} \qquad Q_2 = Q_3 \quad \text{True (series)}$$

*Check your answers before doing part d.*

d. Choose  $>$ ,  $=$  or  $<$  to relate the following pairs of potential differences ( $V_i$  is the potential difference across the plates of capacitor  $C_i$ ):

$$V_1 > V_2$$

$$V_2 > V_3$$

$$V_1 = V_0$$

$$V_3 < V_0$$

e. Find the equivalent capacitance of the arrangement.

$$C_{2-3} = \frac{1}{\frac{1}{C_2} + \frac{1}{C_3}} = 1.2 \mu\text{F}$$

$$C_{eq} = C_1 + C_{2-3} = 2.2 \mu\text{F}$$

- f. What would be the charge of this equivalent capacitor?

$$Q_{eq} = C_{eq}V_0 = 66 \mu\text{C}$$

- g. Find the charge on  $C_1$ .

$$Q_1 = C_1V_1 = C_1V_0 = 30 \mu\text{C}$$

- h. Find the potential difference  $V_A - V_B$ .

$$V_A - V_B = V_2 = \frac{Q_2}{C_2} = \frac{36 \mu\text{C}}{2 \mu\text{F}} = 18 \text{ V}$$

↑

$$Q_2 = Q_{2-3} = C_{2-3}V_0 = 36 \mu\text{C}$$

- i. What is the total electric energy stored in the system?

$$U = \frac{1}{2}C_{eq}V_0^2 = 990 \mu\text{J}$$

- j. A slab of insulator with dielectric constant  $\kappa > 1$  is introduced between the plates of  $C_1$ . The slab fills all the space between the plates. The energy of the system:

*The potential difference remains the same (fixed by the battery). The capacitance increases. Thus the energy increases.*

- k. Which charges change and which do not when the dielectric is inserted?

Change:  $Q_1$  (fixed potential, increased capacitance, thus increased charge)

Do not change:  $Q_2, Q_3$  (fixed potential and fixed capacitance, no reason to change)