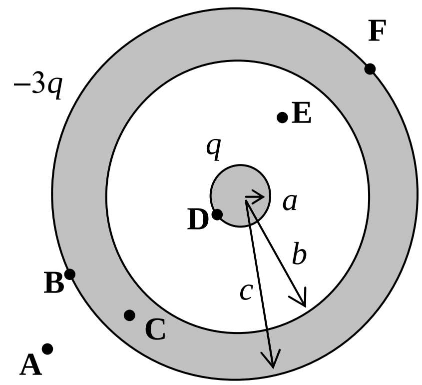
Electric potential.

1. A metallic sphere of radius a is at the center of a metallic shell of radii b and c. The net charge of the sphere is q ($q > 0$). The net charge of the shell is $-3q$. Using Gauss's law, one can prove that:



$E = 0$ $(r < a)$
$E = kq/r^2$ pointing out $(a < r < b)$
$E = 0$ $(b < r < c)$
$E = 2kq/r^2$ pointing in $(r > c)$

a. Some qualitative aspects, first: for points A to F, compare the value of the electric potential for different pairs of points (choose one option for each pair):

To answer this, we need to look at the direction of the electric field: the electric potential decreases in the direction of the electric field.

- $V_A < V_B$ $V_A = V_B$ $\mathbf{V_A > V_B}$
- $V_B < V_C$ $\mathbf{V_B = V_C}$ $V_B > V_C$
- $\mathbf{V_C < V_D}$ $V_C = V_D$ $V_C > V_D$
- $V_B < V_F$ $\mathbf{V_B = V_F}$ $V_B > V_F$

Notice that we did not need to fix the zero of the potential!

b. And now the actual calculation. Let us choose the potential to be zero at infinity. Find the potential for different points. Remember:

$$V(r_1) - V(r_2) = -\int_{r_2}^{r_1} \vec{E} \cdot d\vec{l}$$

- For point A.

$$\begin{aligned} V_A &= V(\infty) - \int_{\infty}^{r_A} E_r dr \\ &= V(\infty) - \int_{\infty}^{r_A} \left(-\frac{2kq}{r^2}\right) dr \\ &= 0 - \left[\frac{2kq}{r}\right]_{\infty}^{r_A} \\ &= -\frac{2kq}{r_A} \end{aligned}$$

- For point C

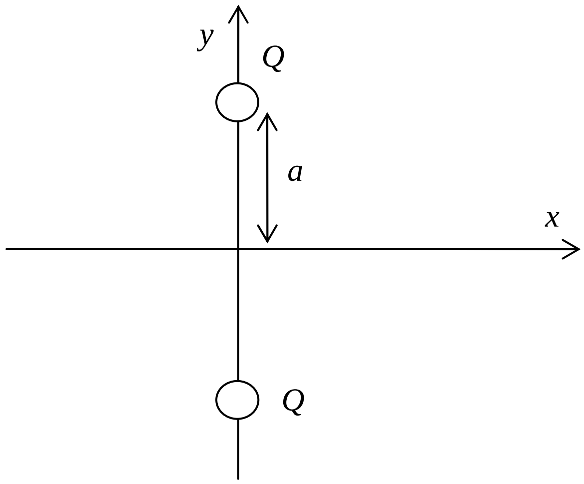
All points in the shell have the same potential, i.e., the potential of the external surface:

$$V_C = -\frac{2kq}{c}$$

- For point E

$$\begin{aligned}
V_A &= V(r=b) - \int_b^{r_E} \frac{kq}{r^2}\,dr \\
&= -\frac{2kq}{c} - \left[-\frac{kq}{r}\right]_b^{r_E} \\
&= -\frac{2kq}{c} + \frac{kq}{r_E} - \frac{kq}{b} \\
&= kq\left(\frac{1}{r_E} - \frac{1}{b} - \frac{2}{c}\right)
\end{aligned}$$

2. Two identical point charges Q are kept fixed on the y axis, at $y = \pm a$ (see figure below). A third, non-fixed charge Q is gently placed right at the origin and released.

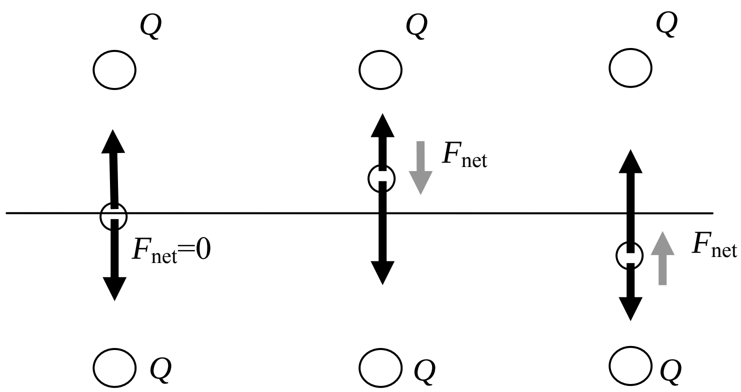


$Q = 2.0\ \mu\text{C}$
$a = 3.0$ cm

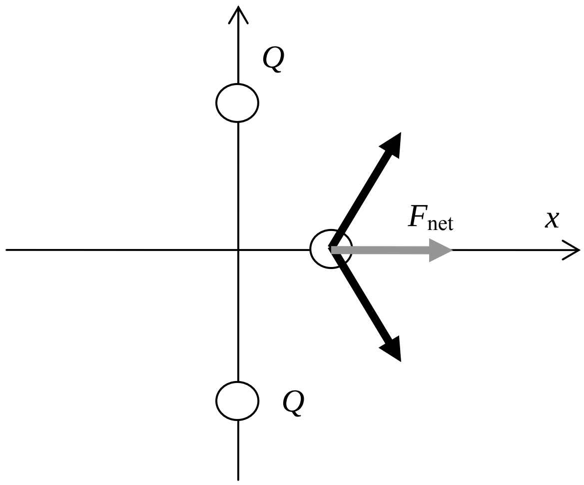
a. What happens to the charge at the origin? Analyze what would happen if the charge was slightly moved in the x direction or in the y direction.

If the charge is exactly at the origin, the net force on it is zero, so it is an equilibrium point.

If the charge moves a little in the positive (negative) y direction, the net force points in the negative (positive) y direction, which pulls the charge back to the equilibrium point. so this looks like stable equilibrium.

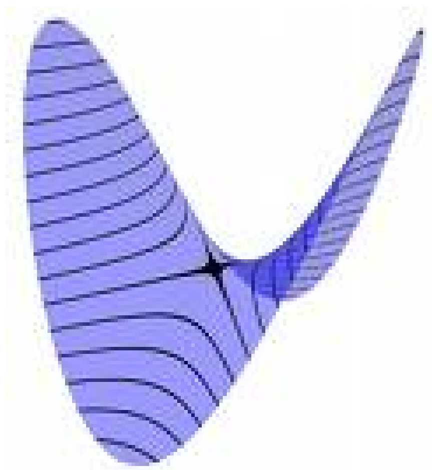


If the charge moves in the x direction, however, the net force pushed it away from the equilibrium point:



This indicates unstable equilibrium.

Overall, this means that the potential energy has a minimum in the y direction and a maximum in the x direction. x = 0 is a "saddle point" (because of the shape of the potential energy surface: something like this



b. The third charge is moved very slightly to the right, along the positive x axis, and released. Find its kinetic energy at $x = a$.

Because the x direction is unstable, the particle will keep moving in the +x direction. Mechanical energy is conserved:

$$U(x=0)+K_0=U(x=a)+K_f$$

$$K_f=K_0+U(x=0)-U(x=a)$$

$$=0+2k\frac{Q^2}{a}-2k\frac{Q^2}{a\sqrt{2}}$$

(Note that when the moving charge is at x = a, its distance to any of the other charges is $a\sqrt{2}$ *)*

$$K_f=2k\frac{Q^2}{a}\left(1-\frac{1}{\sqrt{2}}\right)$$

$$=2\left(9.0\times10^9\ \frac{\text{N m}^2}{\text{C}^2}\right)\frac{\left(2.0\times10^{-6}\ \text{C}\right)^2}{0.030\ \text{m}}\left(1-\frac{1}{\sqrt{2}}\right)$$

$$=0.70\ \text{J}$$