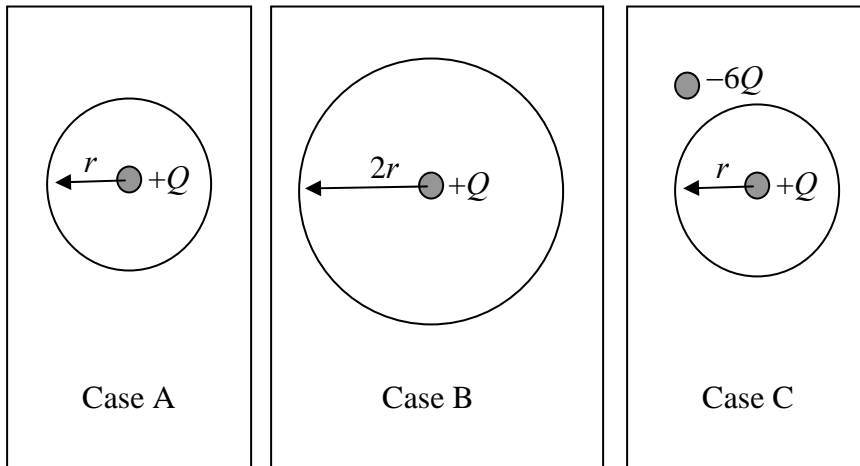


## Electric flux and Gauss' law.

1. Three spherical Gaussian surfaces enclose a charge  $+Q$ . In case C, there is another charge  $-6Q$  outside the surface.



Consider the following conversation:

Student 1: “Since each Gaussian surface encloses the same charge, the net flux through each must be the same.”

Student 2: “Gauss’s law doesn’t apply here. The electric field at the Gaussian surface in case B is weaker than in case A, because the surface is farther from the charge. Since the flux is proportional to the electric field strength, the flux must be smaller in case B.”

Student 3: “I was comparing A and C. In C the charge outside changes the field over the whole surface. The areas are the same, so the flux must be different.”

Do you agree with any of the students? For the students that you think are wrong, what part of their reasoning is incorrect?

*Student 1 is right.*

*Gauss’s law applies anytime you have a closed surface. Student 2 is right about the electric field being weaker on the surface in case B, but the surface area is also larger, and both effects happen to cancel out (area goes as  $r^2$  and electric field as  $r^{-2}$ ).*

*Student C is right about the change in the electric field on the surface: the outer charge does change the electric field. But the flux remains the same (i.e., the changes in the electric field produce the same amount of positive and negative flux).*

2. Shown below is cross-section view of an infinite insulating cylinder of radius  $a$  and an infinite, coaxial metallic cylindrical shell of radii  $b$  and  $c$ . The inner cylinder has a uniform charge per unit length  $\lambda$ . The net charge per unit length of the shell is  $-3\lambda$ .

Find the electric field for different values of  $r$ , the distance from the axis of the system.

a. For  $r > c$ .

*The system has a cylindrical symmetry. Therefore, the electric field must point in the radial direction.*

*Use a cylinder of radius  $r$  and length  $h$  as your Gaussian surface. The flux through it is:*

$$\begin{aligned}\Phi &= \Phi_{\text{caps}} + \Phi_{\text{side}} \\ &= 0 + \Phi_{\text{side}} \\ &= E_r 2\pi r h\end{aligned}$$

*Or, using Gauss's law:*

$$\Phi = \frac{q_{\text{enclosed}}}{\epsilon_0}$$

*In a section of length  $h$ , the charge is  $\lambda h$ . In the same way, a section of the outer shell of length  $h$  has charge  $-3\lambda h$ . Thus*

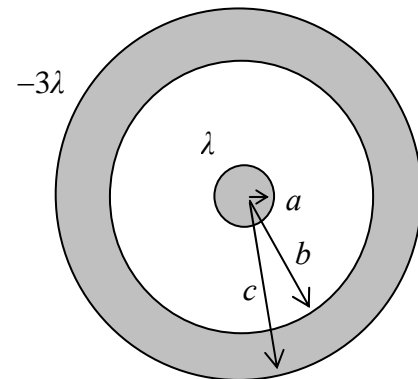
$$\Phi = \frac{\lambda h - 3\lambda h}{\epsilon_0} = -\frac{2\lambda h}{\epsilon_0}$$

Putting everything together:

$$E_r 2\pi r h = -\frac{2\lambda h}{\epsilon_0}$$

$$\boxed{E_r = -\frac{\lambda}{\pi\epsilon_0 r}}$$

*The minus sign indicates that the electric field points inwards.*



- b. For  $b < r < c$ .

*This is inside a conductor, so  $E = 0$ .*

- c. For  $a < r < b$ .

*The process is very similar to part a, except now the enclosed charge comes only from the inner cylinder.*

$$\begin{aligned}\Phi &= \Phi_{\text{caps}} + \Phi_{\text{side}} \\ &= 0 + \Phi_{\text{side}} \quad (\text{with a different } r, \text{ now}) \\ &= E_r 2\pi r h\end{aligned}$$

$$\Phi = \frac{\lambda h}{\epsilon_0} = \frac{\lambda h}{\epsilon_0}$$

$$E_r 2\pi r h = \frac{\lambda h}{\epsilon_0}$$

$$\boxed{E_r = \frac{\lambda}{2\pi\epsilon_0 r}}$$

- d. For  $r < a$  (assume that the charge is uniformly distributed throughout the volume).

*Again, the Gaussian surface is a cylinder, but it is inside the inner cylinder, so we have to be careful with the enclosed charge.*

$$\begin{aligned}\Phi &= \Phi_{\text{caps}} + \Phi_{\text{side}} \\ &= 0 + \Phi_{\text{side}} \quad (\text{again with a different } r) \\ &= E_r 2\pi r h\end{aligned}$$

*We are looking at the charge enclosed in a cylinder of length  $h$  and radius  $r$ , thus with volume  $\pi r^2 h$ . The volume charge density for the inner cylinder is  $\rho = \frac{\lambda}{\pi a^2}$ .*

*Thus the charge in the Gaussian cylinder is*

$$q_r = \pi r^2 h \rho = \lambda h \frac{r^2}{a^2}$$

$$\text{Thus } \Phi = \frac{1}{\epsilon_0} \lambda h \frac{r^2}{a^2}$$

Putting it all together,

$$E_r 2\pi r h = \frac{1}{\epsilon_0} \lambda h \frac{r^2}{a^2}$$

$$E_r = \frac{\lambda}{2\pi\epsilon_0 a^2} r$$

- e. Find the linear charge density on the internal surface of the shell.

*The electric field inside the shell is zero, so if we take the flux through any surface contained in the shell (like a cylinder of length  $h$  and radius between  $b$  and  $c$ ), it must be zero. Thus the charge enclosed by this surface must be zero. Therefore the charge per unit length on the inner surface of the shell must be equal to minus the charge per unit length in the inner cylinder,*

$$\lambda_{\text{inner}} = -\lambda$$

- f. Find the linear charge density on the external surface of the shell.

*The charge on the shell is either on the inner or on the outer surfaces because it is a conductor. The net charge per unit length of the shell is  $-3\lambda$ , and the inner surface has  $-\lambda$ , so the outer surface must have  $-2\lambda$ .*