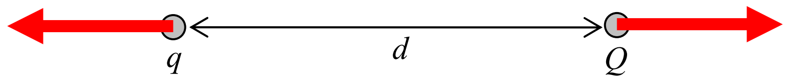
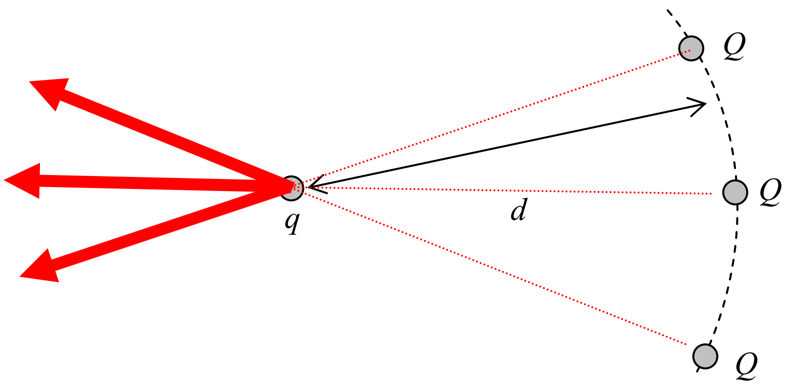
Coulomb's force. Electric field.

1. Two positive charges *q* and *Q* (*Q* > *q*) are held in place a distance *d* apart. Draw the electrostatic force exerted on each charge by the other.



Two extra charges are added to the system as shown below.



Two students discuss the force on charge *q*:

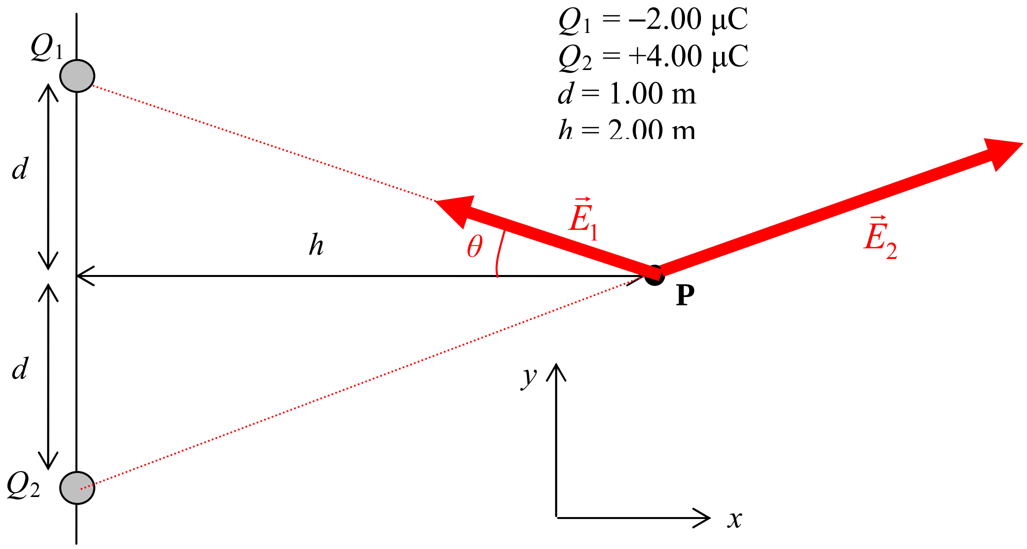
Student 1: *The net electric force on charge q is now three times as large as before, since there are now three positive charges exerting forces on it.*

Student 2: *I don't think so. The force from the Q charge on top will cancel with the force from the Q charge at the bottom, so the net electric force will be the same as before.*

Do you agree with either student? Explain.

They are both wrong.
Student 1 is forgetting that electric forces are vectors, so one cannot simply add the magnitudes of the forces. Direction matters.
Student 2 seems to be more aware of the fact that forces are vectors, because he knows that two "opposing" forces can cancel out. But in this case, only the vertical components of the forces due to the top and bottom charges cancel out. The horizontal components add up.

2. Two charges Q_1 and Q_2 are placed at fixed positions on a vertical axis as shown in the figure below.



Find the net electric field (magnitude and direction) at point P.

The electric field due to each of the two charges has magnitudes:

$$E_1 = k_e \frac{|Q_1|}{d^2 + h^2} \qquad E_2 = k_e \frac{|Q_2|}{d^2 + h^2}$$

We need to take components and add them separately in the x and y directions.

$$E_x = -E_1 \cos\theta + E_2 \cos\theta$$
$$E_y = E_1 \sin\theta + E_2 \sin\theta$$

with

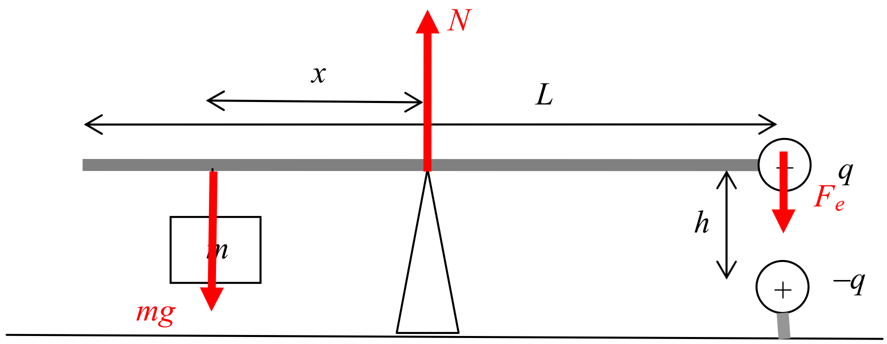
$$\sin\theta = \frac{d}{\sqrt{d^2 + h^2}} \qquad \cos\theta = \frac{h}{\sqrt{d^2 + h^2}}$$

Thus:

$$
\begin{aligned}
E_x &= \left(-E_1 + E_2\right)\cos\theta \\
&= k_e \frac{h}{\left(d^2 + h^2\right)^{3/2}}\left(\left|Q_2\right| - \left|Q_1\right|\right) \\
&= \left(8.99\times10^9\ \frac{\mathrm{Nm^2}}{\mathrm{C^2}}\right)\frac{2.00\mathrm{m}}{\left(1.00^2 + 2.00^2\right)^{3/2}\ \mathrm{m^3}}\left(4.00 - 2.00\right)\times10^{-6}\,\mathrm{C} = 3220\ \mathrm{N/C}
\end{aligned}
$$

$$
\begin{aligned}
E_y &= \left(E_1 + E_2\right)\sin\theta \\
&= k_e \frac{d}{\left(d^2 + h^2\right)^{3/2}}\left(\left|Q_2\right| + \left|Q_1\right|\right) \\
&= \left(8.99\times10^9\ \frac{\mathrm{Nm^2}}{\mathrm{C^2}}\right)\frac{1.00\mathrm{m}}{\left(1.00^2 + 2.00^2\right)^{3/2}\ \mathrm{m^3}}\left(4.00 + 2.00\right)\times10^{-6}\,\mathrm{C} = 4820\ \mathrm{N/C}
\end{aligned}
$$

3. A long, non-conducting massless rod of length L is pivoted at its center and balanced with a block of mass m at a distance x from the pivot. A small sphere with positive charge q is attached to the opposite end. Another charge $-q$ is right under these sphere, at distance h. Find h as a function of the other quantities when the rod is horizontal and balanced.



Net torque about the pivot point must be zero:

$$mgx - F_e \frac{L}{2} = 0 \qquad \text{with } F_e = k\frac{q^2}{h^2}$$

$$mgx = k\frac{q^2}{h^2}\frac{L}{2}$$

$$h = \sqrt{\frac{kq^2 L}{2mgx}}$$