Fluids: dynamics.

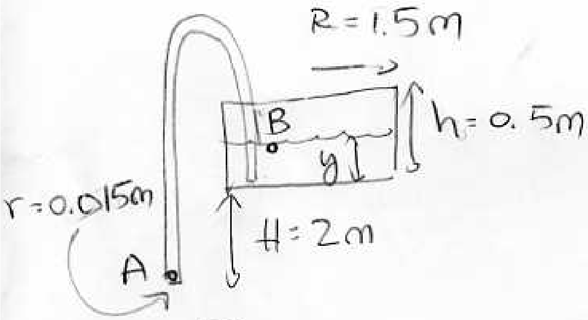
Backyard pools often do not have a drain at the bottom, so people are supposed to empty them by tipping them over, which is not an easy task when the pool is filled with water. A typical children's pool may have a circular base of about 3 m in diameter and a depth of 50 cm.

a) What is the mass of the water in the pool when the pool is completely full?

$$m = \left(1000\ \frac{kg}{m^3}\right)\pi(1.5\,m)^2(0.5\,m) = \boxed{3530\ kg}$$

So that's why tipping it over is not a good option. But you can build a siphon with a simple garden hose.

b) The diameter of the hose is 3 cm. You are lucky to have a slope near your pool, so that you can drain the water to a point that is *H* = 2 m below the bottom of the pool. The intake end of the hose is at the bottom of the pool. Determine the draining rate (in liters per second) as a function of *y*, the height of water left in the pool.



Bernoulli for A, B:

$$P_A + \frac{1}{2}\rho v_A^2 - \rho g H = P_A + \frac{1}{2}\rho v_B^2 + \rho g y$$

Continuity for A, B:

$$A_A v_A = A_B v_B$$

$$P_A = P_B = P_{atm}$$

$$\frac{1}{2}\cancel{\rho}(v_A^2 - v_B^2) = \cancel{\rho} g(H+y)$$

$$v_B = \frac{A_A}{A_B} v_A = \left(\frac{r}{R}\right)^2 v_A$$

$$v_A^2\left[1-\left(\frac{r}{R}\right)^4\right] = 2g(H+y) \qquad v_A = \sqrt{\frac{2g(H+y)}{1-\left(\frac{r}{R}\right)^4}}$$

$$\frac{dV}{dt} = A_A v_A = \boxed{\pi r^2 \sqrt{\frac{2g(H+y)}{1-\left(\frac{r}{R}\right)^4}}}$$

(Note: Because $\frac{r}{R} = 10^{-2}$, $1-\left(\frac{r}{R}\right)^4 \simeq 1$ ie we could have taken $v_B \simeq 0$ in Bernoulli)

c) Why is it important to have a slope by the pool? (Hint: Examine the draining rate when the pool is almost empty.)

If $H = 0$ (no slope), the rate goes to zero when $y \to 0$ (ie, it is impossible to empty the pool completely)

d) How long will it take to empty the pool?

$$v_B = -\frac{dy}{dt} \qquad v_B = \frac{A_A}{A_B} v_A = \left(\frac{r}{R}\right)^2 v_A$$

$$-\frac{dy}{dt} = \left(\frac{r}{R}\right)^2 \sqrt{\frac{2g(H+y)}{1-\left(\frac{r}{R}\right)^4}}$$ Differential equation

$$\left(\frac{R}{r}\right)^2 \sqrt{\frac{1-\left(\frac{r}{R}\right)^4}{2g}} \int_h^0 \frac{dy}{\sqrt{H+y}} = -\int_0^t dt$$

$$t = -\left(\frac{R}{r}\right)^2 \sqrt{\frac{1-\left(\frac{r}{R}\right)^4}{2g}}\, 2\left(\sqrt{H} - \sqrt{H+h}\right) \qquad \frac{R}{r} = 10^2$$

$$t = 10^4 \sqrt{\frac{1-10^{-8}}{2(9.81\frac{m}{s^2})}}\, 2\left(\sqrt{12.5m} - \sqrt{2m}\right) = 754\,s = \boxed{12.6\,min}$$