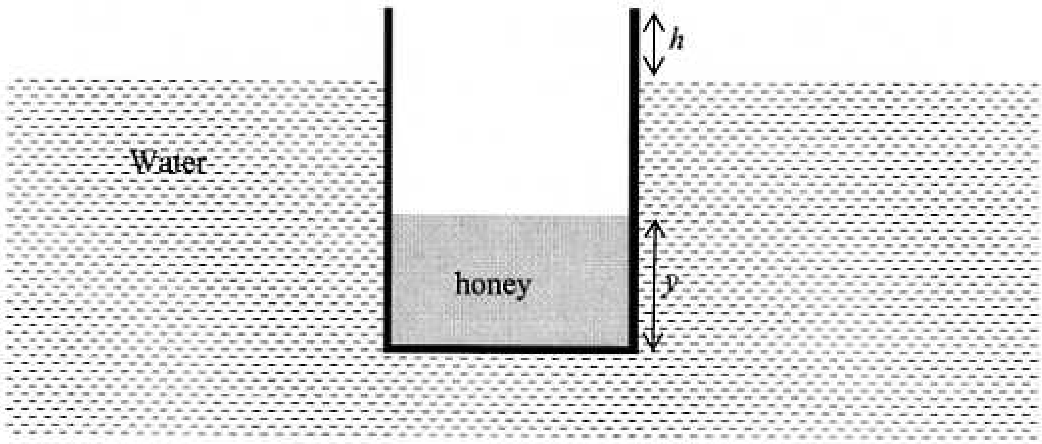
Fluids: Statics

1. An empty cylindrical container with thin walls (base area $A = 20\text{cm}^2$ and height $H = 30$ cm) is placed on the water. Equilibrium is achieved when half of the empty container is under the water surface. Honey (density = 1420 g/liter) is then slowly poured inside the container.



   a. What is the mass of the container?

$$mg = V_{submerged}\,\rho_w\, g$$

$$m = V_{sub}\,\rho_w = (20\times10^{-4}\,m^2)(0.15\,m)(1000\,\tfrac{kg}{m^3}) = \boxed{0.3\,kg}$$

   b. Determine h, the height of the part of the container that remains above the water surface as a function of y, the height of the honey inside the container. (Check your result: what mass of honey is needed to sink the container?)

$$mg + m_h g = V_s \rho_w g$$

$$m + \rho_h A y = A(H-h)\rho_w$$

$$h = H - \frac{m + \rho_h A y}{A\rho_w}$$

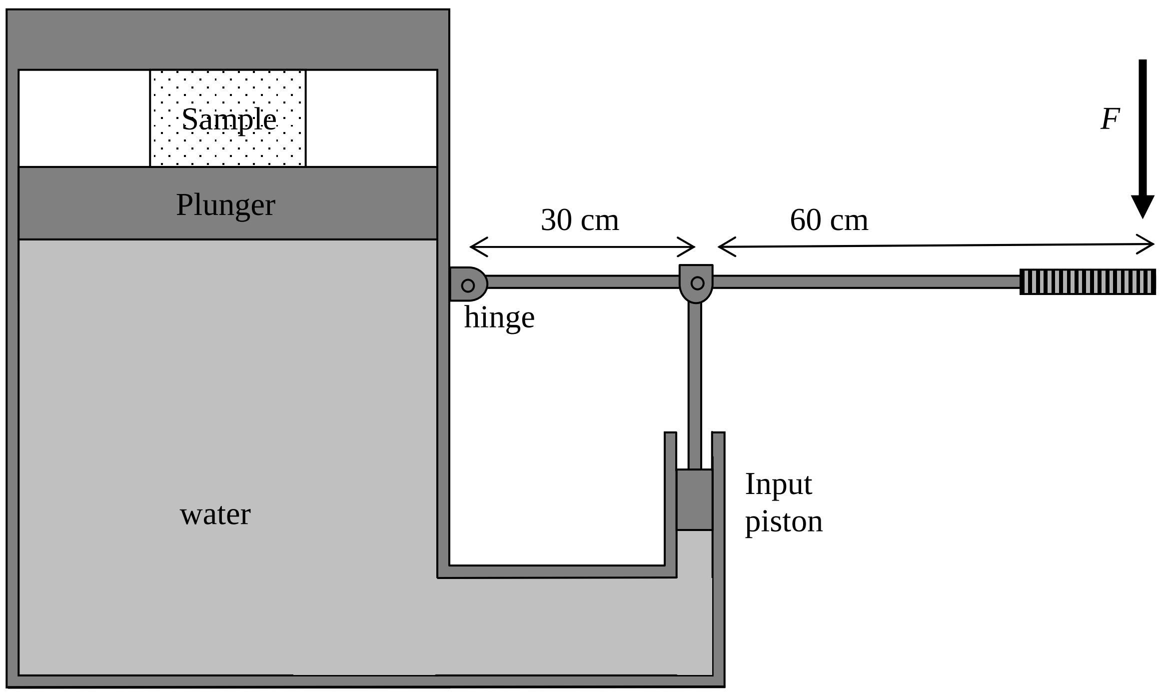
$$h = 0.3\,m - \frac{0.3\,kg + (1420\,\frac{kg}{m^3})(20\times10^{-4}\,m^2)\,y}{(20\times10^{-4}\,m^2)(1000\,kg/m^3)}$$

$$\boxed{h = (0.15 - 1.42y)\,m}$$

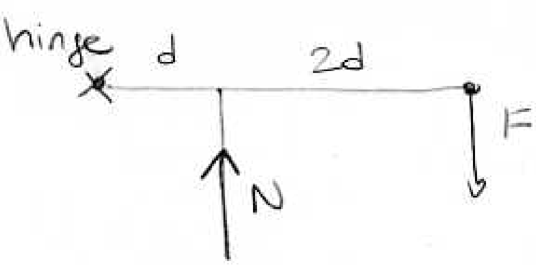
Check: For $h = 0$, $y = \frac{0.15}{1.42} = 0.11\,m$

$m_{honey} = (1420\,\frac{kg}{m^3})(20\times10^{-4}\,m^2)(0.11\,m) = 0.3\,kg$ (of course!)

2. The hydraulic press shown below is operated by pressing on the lever that is hinged to a fixed point and connected to the input piston. The input piston has an area of 2.00 cm^2, and the area of the plunger is 100 times larger. The press is used to compress a sample of steel (a cube of side 10.0 cm) by exerting a force *F* = 300 N at the end of the lever (see figure). By how much is the height of the sample reduced? (Young's modulus for steel = 200 GPa). Hint: You can assume that the input piston does not move significantly.



First we need to find the force exerted on the piston.



Torque about hinge:

$$dN - 3dF = 0$$

$$N = 3F = 900\text{ N}$$

Pascal's principle: $\dfrac{F_{out}}{A_{out}} = \dfrac{N}{A_{in}}$

$$F_{out} = N\frac{A_{out}}{A_{in}} = N \times 100 = 9\times10^{4}\text{ N}$$

$L_0 = d = 10$ cm

$A_{cube} = d^2$

$$\Delta L = L_0 \frac{F_{our}/A_{cube}}{Y}$$

$$= \cancel{d} \frac{F_{our}}{d^{\cancel{2}} Y} = \frac{9 \times 10^4 \text{ N}}{(0.10 \text{ m})(200 \times 10^9 \text{ Pa})}$$

$$= 4.5 \times 10^{-6} \text{ m}$$

$$= \boxed{4.5 \ \mu\text{m}}$$