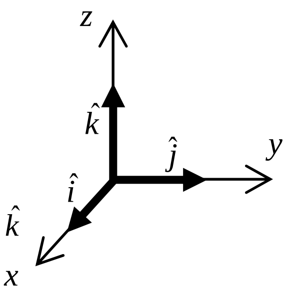
Formula Sheet – Phys 222– Fall 2009



Vectors and math

$$\left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2 + A_z^2} \qquad \vec{A}\cdot\vec{B} = AB\cos\theta = A_xB_x + A_yB_y + A_zB_z$$

$$\left|\vec{A}\times\vec{B}\right| = AB\sin\theta \qquad \vec{A}\times\vec{B} = \left(A_yB_z - A_zB_y\right)\hat{i} + \left(A_zB_x - A_xB_z\right)\hat{j} + \left(A_xB_y - A_yB_x\right)\hat{k}$$

$$ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{d}{dx}x^n = nx^{n-1} \qquad \frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

10^{-15}	femto- (f)
10^{-12}	pico- (p)
10^{-9}	nano- (n)
10^{-6}	micro- (μ)
10^{-3}	milli- (m)
10^{-2}	centi- (c)
10^{3}	kilo- (k)
10^{6}	mega- (M)
10^{9}	giga- (G)
10^{12}	tera- (T)

Geometry

perimeter circle: $2\pi R$ area sphere: $4\pi R^2$

area circle: πR^2 volume sphere: $\frac{4}{3}\pi R^3$

1 revolution = 2π radians = 360°

Conversion factors

$1\ \text{m}^3 = 1000$ liters $\qquad$ 1 atm = 1.01×10^5 Pa = 760 mm Hg

Physical constants

$$g = 9.81\ \text{m/s}^2 \qquad |e| = 1.6\times10^{-19}\ \text{C}$$

$$\mu_0 = 4\pi\times10^{-7}\ \frac{\text{Wb}}{\text{A}\cdot\text{m}} \qquad \varepsilon_0 = 8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2} \qquad c = 3.00\times10^8\ \text{m/s}$$

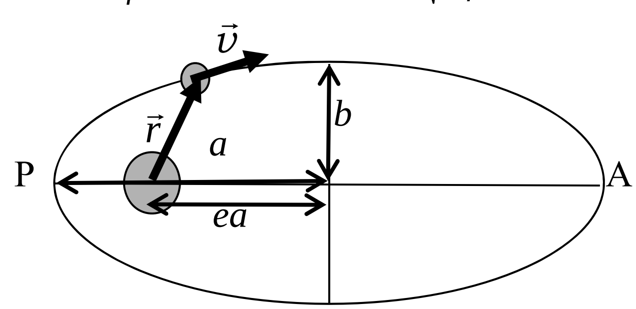
Fluids

$$P = \frac{F}{A} \qquad P = P_0 - \rho g y \qquad \frac{dV}{dt} = A\upsilon = \text{constant} \qquad P + \rho g y + \frac{1}{2}\rho\upsilon^2 = \text{constant}$$

Gravitation

$$\left|\vec{F}_{\text{Newton}}\right| = G\frac{Mm}{r^2} \qquad g = G\frac{M}{r^2} \qquad U = -G\frac{Mm}{r} \qquad \upsilon_{\text{circular orbit}} = \sqrt{\frac{GM}{r}}$$

$$T = \frac{2\pi a^{3/2}}{\sqrt{GM}} \qquad \vec{r}\times\vec{\upsilon} = \text{constant} \qquad r_A\upsilon_A = r_P\upsilon_P$$



$$g = 9.81\ \text{m/s}^2 \qquad G = 6.67\times10^{-11}\ \text{Nm}^2/\text{kg}^2$$

$$M_E = 5.97\times10^{24}\ \text{kg} \qquad R_E = 6.38\times10^6\ \text{m}$$

Electrostatics

$$\vec{F}_{\text{Coulomb}} = k_e \frac{q_1 q_2}{r^2}\hat{r} \qquad \vec{E} = \frac{\vec{F}}{q_0} \qquad \vec{p} = |Q|\vec{d} \qquad \vec{\tau} = \vec{p}\times\vec{E} \qquad U = -\vec{p}\cdot\vec{E}$$

$$\Phi_E = \int_{\text{surface}} \vec{E}\cdot d\vec{a} \qquad \int_{\substack{\text{closed}\\ \text{surface}}} \vec{E}\cdot d\vec{a} = \frac{q_{\text{enclosed}}}{\epsilon_0} \qquad \vec{E} = k_e \frac{q}{r^2}\hat{r} \qquad \vec{E} = k_e \frac{2\lambda}{r}\hat{r} \qquad \vec{E} = \pm\frac{\sigma}{2\epsilon_0}\hat{x}$$

$$\rho = \frac{dQ}{dV} \quad (V \equiv \text{volume}) \qquad \sigma = \frac{dQ}{dA} \quad (A \equiv \text{area}) \qquad \lambda = \frac{dQ}{dl} \quad (l \equiv \text{length})$$

$$V_A - V_B = -\int_B^A \vec{E}\cdot d\vec{l} \qquad \vec{E} = -\vec{\nabla}V \qquad W = -\Delta U = -q_0 \Delta V \qquad U = q_0 V$$

$$V = k_e \frac{q}{r} \qquad \Delta V = \pm Ed$$

$$k_e = \frac{1}{4\pi\epsilon_0} = 8.99\times 10^{9}\ \frac{\text{N m}^2}{\text{C}^2} \qquad \epsilon_0 = 8.85\times 10^{-12}\ \frac{\text{C}^2}{\text{N m}^2} \qquad e = 1.60\times 10^{-19}\ \text{C}$$

$$m_{\text{e}} = 9.11\times 10^{-31}\ \text{kg} \qquad m_{\text{p}} = 1.67\times 10^{-27}\ \text{kg}$$

Capacitors

$$C = \frac{Q}{V} \qquad C_{\text{eq}} = C_1 + C_2 + C_3 + \ldots \qquad \frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

$$C = \epsilon_0 \frac{A}{d} \qquad C = \frac{2\pi\epsilon_0 L}{\ln(a/b)} \qquad C = 4\pi\epsilon_0 \frac{ab}{b-a} \qquad C = 4\pi\epsilon_0 R$$

$$U = \frac{1}{2}CV^2 = \frac{Q^2}{2C} = \frac{1}{2}QV \qquad u = \frac{1}{2}\epsilon E^2 \qquad \epsilon = \kappa\epsilon_0 \qquad C = \kappa C_0$$

Electric current and resistor circuits

$$I = \frac{dQ}{dt} \qquad V = IR \qquad J = \frac{I}{A} = qnv_d \qquad E = J\rho \qquad R = \rho\frac{L}{A}$$

$$\rho = \rho_0\left(1 + \alpha\left(T - T_0\right)\right) \qquad R = R_0\left(1 + \alpha\left(T - T_0\right)\right)$$

$$R_{\text{eq}} = R_1 + R_2 + R_3 + \ldots \qquad \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \qquad P = IV = I^2R = \frac{V^2}{R}$$

***RC* circuits**

$$Q(t) = Q(\infty)\left(1 - e^{-t/\tau}\right) \qquad Q(t) = Q(0)e^{-t/\tau} \qquad \tau = RC \qquad I(t) = I(0)e^{-t/\tau}$$

Magnetic field

$$\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right) \qquad d\vec{F} = Id\vec{l}\times\vec{B} \qquad \Phi_B = \int \vec{B}\cdot d\vec{A} \qquad R = \frac{mv}{|q|B}$$

$$\mu = IA \qquad \vec{\tau} = \vec{\mu}\times\vec{B} \qquad U = -\vec{\mu}\cdot\vec{B}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} \qquad d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2} \qquad B = \frac{\mu_0 I}{2\pi x} \qquad \frac{F}{L} = \frac{\mu_0 II'}{2\pi r} \qquad c^2 = \frac{1}{\mu_0\varepsilon_0}$$

$$B_x = \frac{\mu_0 I a^2}{2\left(x^2+a^2\right)^{3/2}} \qquad B_x = \frac{\mu_0 NI}{2a} \qquad B = \mu_0 nI \qquad \oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

Induction

$$\varepsilon = -N\frac{d\Phi_B}{dt} \qquad \varepsilon = \oint\left(\vec{v}\times\vec{B}\right)\cdot d\vec{l} \qquad \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(i_C + i_D\right)_{\text{enclosed}} \qquad i_D = \varepsilon\frac{d\Phi_E}{dt}$$

Maxwell's equations

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0} \qquad \oint \vec{B}\cdot d\vec{A} = 0$$

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B}\cdot d\vec{l} = \mu_0\left(i_C + \varepsilon_0\frac{d\Phi_E}{dt}\right)_{\text{enclosed}}$$

Inductance

$$\varepsilon_1 = -M\frac{di_2}{dt} \qquad \varepsilon_2 = -M\frac{di_1}{dt} \qquad M = \frac{N_1\Phi_{B1}}{i_2} = \frac{N_2\Phi_{B2}}{i_1} \qquad \varepsilon = -L\frac{di}{dt} \qquad L = \frac{N\Phi_B}{i}$$

$$i = I_\infty\left(1 - e^{-\frac{t}{\tau}}\right) \qquad i = I_0 e^{-\frac{t}{\tau}} \qquad \tau = \frac{L}{R} \qquad L = \mu_0 n^2 lA \qquad U = \frac{1}{2}LI^2 \qquad u = \frac{B^2}{2\mu_0}$$

$$q = Q_0\cos\left(\omega t + \phi\right) \qquad \omega = \sqrt{\frac{1}{LC}}$$

$$q = Q_0 e^{-\frac{R}{2L}t}\cos\left(\omega' t + \phi\right) \qquad \omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

AC current

$$i = I\cos\omega t \qquad I_{\text{rav}} = \frac{2}{\pi}I \qquad I_{\text{rms}} = \frac{I}{\sqrt{2}} \qquad V_{\text{rms}} = \frac{V}{\sqrt{2}} \qquad V_{\text{rms}} = I_{\text{rms}}Z$$

$$V_R = IR \qquad V_L = IX_L \qquad X_L = L\omega \qquad V_C = IX_C \qquad X_C = \frac{1}{C\omega}$$

$$V = IZ \qquad Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} \qquad \tan\phi = \frac{X_L - X_C}{R} \qquad v = V\cos\left(\omega t + \phi\right)$$

$$p = iv \qquad P_{\text{average}} = \frac{1}{2}IV\cos\phi = I_{\text{rms}}V_{\text{rms}}\cos\phi = I_{\text{rms}}^2 R \qquad \frac{V_2}{V_1} = \frac{N_2}{N_1} \qquad V_1I_1 = V_2I_2$$

EM waves

$$\vec{E}\left(x,t\right) = E_{\max}\cos\left(kx - \omega t\right)\hat{j} \qquad \vec{B}\left(x,t\right) = B_{\max}\cos\left(kx - \omega t\right)\hat{k} \qquad E_{\max} = cB_{\max}$$

$$E = cB \qquad c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \qquad u = \frac{1}{2}\varepsilon_0E^2 + \frac{1}{2\mu_0}B^2 = \varepsilon_0E^2 = \frac{B^2}{\mu_0} \qquad \frac{1}{A}\frac{dp}{dt} = \frac{S}{c} = \frac{EB}{\mu_0 c}$$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \qquad I = S_{\text{average}} = \frac{E_{\max}B_{\max}}{2\mu_0} = \frac{E_{\max}^2}{2c\mu_0} = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}E_{\max}^2 = \frac{1}{2}\varepsilon_0 cE_{\max}^2$$

Geometric optics

$$n = \frac{c}{v} \qquad \theta_i = \theta_r \qquad n_1\sin\theta_1 = n_2\sin\theta_2 \qquad \sin\theta_{crit} = \frac{n_2}{n_1} \qquad I = I_{\max}\cos^2\phi \qquad \tan\theta_P = \frac{n_2}{n_1}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad m = \frac{y'}{y} = -\frac{s'}{s} \qquad f_{\text{spherical mirror}} = \frac{R}{2} \qquad \frac{1}{f} = \frac{n_{\text{in}} - n_{\text{out}}}{n_{\text{out}}}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \qquad m = \frac{y'}{y} = -\frac{n_a s'}{n_b s} \qquad M = \frac{\theta'}{\theta}$$

Interference and diffraction

$$d\sin\theta = m\lambda \qquad y = mR\frac{\lambda}{d} \qquad I = I_{\max}\cos^2\left(\frac{\phi}{2}\right) \qquad \phi = 2\pi\frac{r_2 - r_1}{\lambda} = 2\pi\frac{d\sin\theta}{\lambda}$$

$$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda \qquad 2t = m\lambda_n \qquad 2t = \left(m + \frac{1}{2}\right)\lambda_n$$

$$a\sin\theta = m\lambda \qquad \beta = 2\pi\frac{a\sin\theta}{\lambda} \qquad I = I_{\max}\frac{\sin^2\left(\frac{\beta}{2}\right)}{\left(\frac{\beta}{2}\right)^2} \qquad I = I_{\max}\cos^2\left(\frac{\phi}{2}\right)\frac{\sin^2\left(\frac{\beta}{2}\right)}{\left(\frac{\beta}{2}\right)^2}$$

$$R = \frac{\lambda}{\Delta\lambda} = Nm \qquad \sin\theta_1 = 1.22\frac{\lambda}{D} \qquad 2d\sin\theta = m\lambda$$