Lecture 41

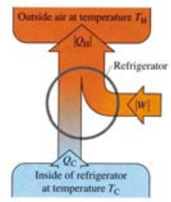
Second law of thermodynamics.
Carnot cycle. Absolute zero.

ACT: Leaving the fridge open

If you leave the door of your fridge open, you will get a heart-stopping electricity bill, but you will also:

A. Freeze the kitchen

B. Warm up the kitchen



Now the fridge and the kitchen are one system. We are taking Q_C out of this system and dumping Q_H into it. Overall, heat is added!

$$|Q_H| = |Q_C| + W \quad > |Q_C|$$

Second law of thermodynamics

It is impossible for any system to undergo a process in which it absorbs heat form a reservoir at a single temperature and convert the heat completely into mechanical work, with the system ending in the same state as it began ("engine" or Kelvin-Plank statement)

i.e.,

It is impossible to build a 100%-efficient heat engine ($e = 1$)

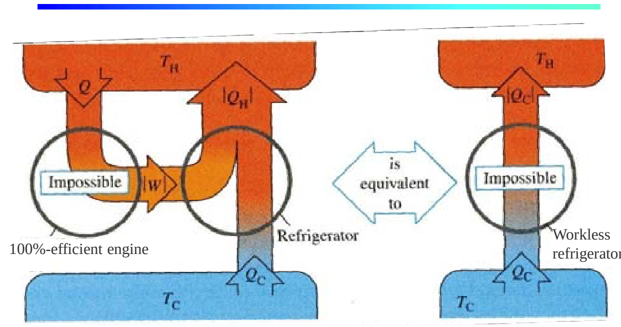
Or

It is impossible for any process to have as its sole result the transfer of heat from a cooler to a hotter body ("refrigerator" or Clausius statement)

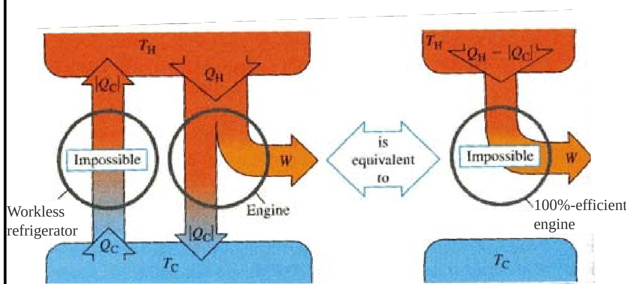
i.e.,

It is impossible to build a workless refrigerator ($K \to \infty$)

Clausius ⇒ Kelvin-Plank

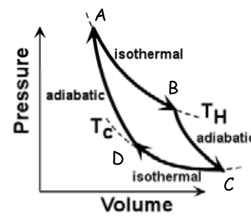


Kelvin-Plank ⇒ Clausius



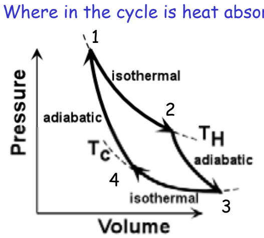
The Carnot cycle

Cycle made with reversible isothermal and adiabatic processes.



ACT: Carnot cycle

Where in the cycle is heat absorbed?



A. At point 1
B. Between 1 and 2
C. Between 4 and 1

To absorb heat, we need a *process* between two states. 1 is a state.

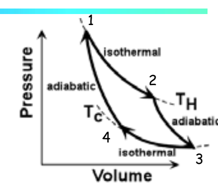
No heat transfer between 4 and 1 (adiabatic)

Between 1 and 2, temperature is always T_H

$$\Delta U_{1\to 2} = 0$$
$$Q_{1\to 2} = W_{1\to 2} > 0$$

Heat exchange in Carnot's cycle

$$\Delta U_{1\to 2} = 0$$
$$Q_{1\to 2} = W_{1\to 2} = nRT_H \ln\frac{V_2}{V_1} > 0$$
$$\Delta U_{3\to 4} = 0$$
$$Q_{3\to 4} = W_{3\to 4} = nRT_C \ln\frac{V_4}{V_3} < 0$$



$$T_H V_1^{\gamma-1} = T_C V_4^{\gamma-1}$$
$$T_H V_2^{\gamma-1} = T_C V_3^{\gamma-1}$$
$$\frac{T_H}{T_C} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \qquad \frac{V_4}{V_1} = \frac{V_3}{V_2} \qquad \frac{V_2}{V_1} = \frac{V_3}{V_4}$$

$$\frac{Q_H}{Q_C} = \frac{Q_{1\to 2}}{Q_{3\to 4}} = \frac{nRT_H \ln\frac{V_2}{V_1}}{nRT_C \ln\frac{V_4}{V_3}} = -\frac{T_H}{T_C}$$

$$\boxed{\frac{Q_H}{Q_C} = -\frac{T_H}{T_C}}$$

Carnot efficiency

$$e = \frac{W}{Q_H} = \frac{Q_C + Q_H}{Q_H} = 1 + \frac{Q_C}{Q_H} = 1 - \frac{T_C}{T_H}$$

$$\boxed{e_{Carnot} = 1 - \frac{T_C}{T_H}}$$

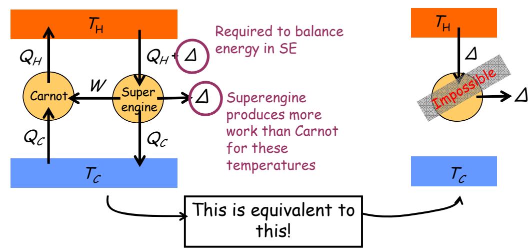
Carnot's cycle is completely reversible. Run backwards, it is a Carnot refrigerator

$$K = \frac{Q_C}{|W|} = \frac{|Q_C|}{|Q_H| - |Q_C|} = \frac{T_C}{T_H - T_C}$$

$$\boxed{K_{Carnot} = \frac{T_C}{T_H - T_C}}$$

Carnot is the ideal cycle

The Carnot cycle is completely reversible. So let's use it as a refrigerator, and couple it to a hypothetical superengine with $e_{SE} > e_{Carnot}$.



No engine can be more efficient than a Carnot engine operating between the same two temperatures.

Absolute zero

The best refrigerator you can get (Carnot) has performance

$$K_{Carnot} = \frac{T_C}{T_H - T_C}$$

The colder you try to go, the less efficient the refrigerator gets:

$$K_{Carnot} = \frac{T_C}{T_H - T_C} \xrightarrow{T_C \to 0} 0$$

Since heat leaks will not disappear as the object is cooled, you need more cooling power the colder it gets.
The integral of the power required diverges as $T \to 0$.

Therefore **you cannot cool a system to absolute zero**