Lecture 40

Heat engines and refrigerators.

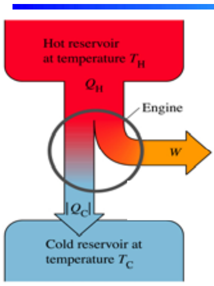
Heat engine

= device with a working substance (eg. gas) that operates in a thermodynamic cycle. In each cycle, the net result is that the system absorbs heat ($Q > 0$) and does work ($W > 0$).

Examples:

- Car engine: burns fuel, heats air inside piston. Piston expands, does mechanical work to move car
- Animal: burns "food" to be able to move

Hot and cold reservoirs

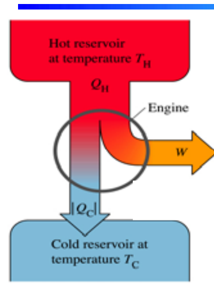


Stages of the cycle

- Absorb heat from hot reservoir (Q_H)
- Perform mechanical work (W)
- Dump excess heat into cold reservoir ($Q_C < 0$)

Reservoir = large body whose temperature does not change when it absorbs or releases heat.

Energy flow



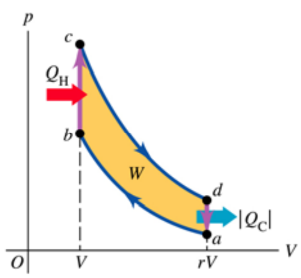
Working substance in engine completes a cycle, so $\Delta U = 0$:

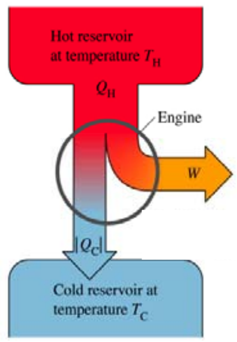
$$(Q_H + Q_C) - W = 0$$

$$\boxed{W = Q_H + Q_C = |Q_H| - |Q_C|}$$

This relation follows naturally from the diagram (Q_H "splits"). Draw it every time!

Energy flow diagrams





Limitations

We are not saying that you can absorb 10 J of heat from a hot source (a burning fuel) and produce 10 J of mechanical work...

You can absorb 10 J of heat from a hot source (a burning fuel) and produce 7 J of mechanical work and release 3 J into a cold source (cooling system).

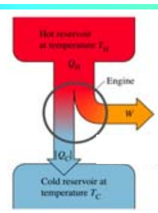
... so at the end you absorbed 10 J but *used* (= converted to work) only 7 J.

(We'll see later that it is impossible to make $Q_H = W$, or $Q_C = 0$)

Efficiency

$$\text{Efficiency} = \frac{\text{what you use}}{\text{what you pay for}}$$

For a heat engine: $e = \frac{W}{Q_H}$ $\quad 0 < e < 1$



Example: A heat engine does 30 J of work and exhausts 70 J by heat transfer. What is the efficiency of the engine?

$W = 30$ J

$|Q_c| = 70$ J $\Rightarrow$ $Q_c = -70$ J

$$e = \frac{W}{Q_H} = 0.3 \quad \text{(or 30\%)}$$

$Q_H = W - Q_c = 100$ J

ACT: Two engines

Two engines 1 and 2 with efficiencies e_1 and e_2 work in series as shown. Let e be the efficiency of the combination. Which of the following is true?

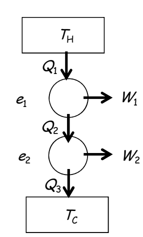
A. $e > e_1 + e_2$

B. $e = e_1 + e_2$

C. $e < e_1 + e_2$

$$e_1 = \frac{W_1}{|Q_1|}$$

$$e_2 = \frac{W_2}{|Q_2|}$$

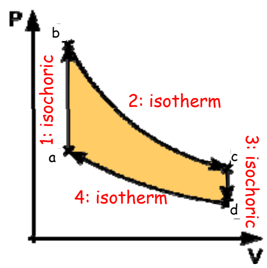


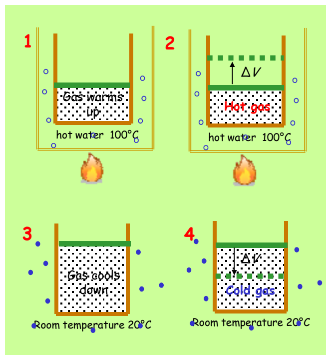
$$e = \frac{W_1 + W_2}{|Q_1|} = \frac{W_1}{|Q_1|} + \frac{W_2}{|Q_1|} < \frac{W_1}{|Q_1|} + \frac{W_2}{|Q_2|}$$

$$|Q_1| = |Q_2| + W_1 > |Q_2| \qquad \frac{1}{|Q_1|} < \frac{1}{|Q_2|}$$

The Stirling engine

DEMO: Stirling engine





Internal combustion engines

The heat source (fuel combustion) is *inside* the engine and mixed with the working substance (air)

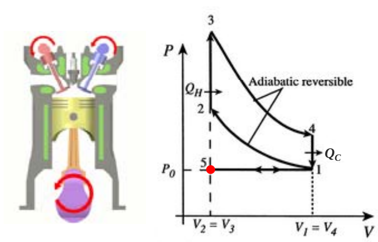
- Otto (4 stroke gasoline)
- Diesel

Note: No real cold and hot reservoirs.

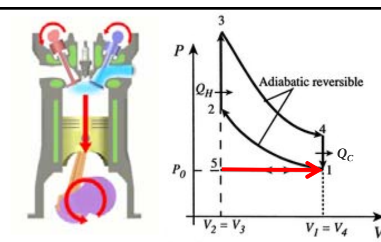
Otto cycle

Idealization of the four-stroke gasoline engine

Start

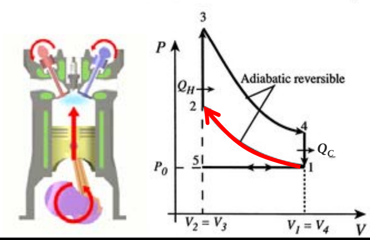


Intake:
- mix of air and fuel enter
- at p_{atm}
- n increase

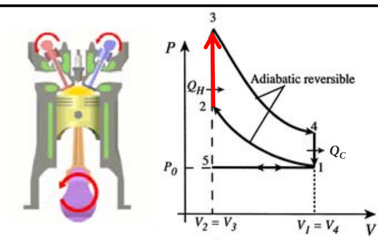


Compression:

Adiabatic compression:
- temperature increase
- no heat exchange
- work done **on** the gas (small because of small pressure)

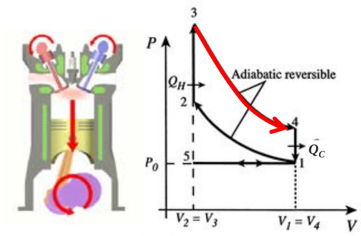


Combustion:
Heating at constant volume
- No work



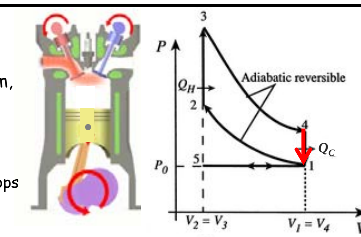
Power stroke:

Adiabatic expansion
- Temperature decrease
- No heat exchange
- Work done by the gas (large because of large pressure)



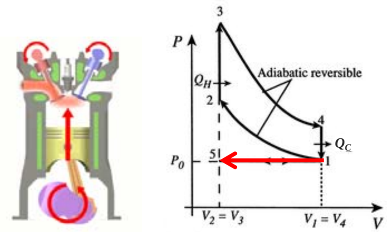
Heat reject:

When piston at the bottom, very fast cooling, i.e. at constant volume
- Excess heat absorbed by water jacket
- Valve opens → Pressure drops to p_{atm}



Exhaust:

n decrease



Compression ratio

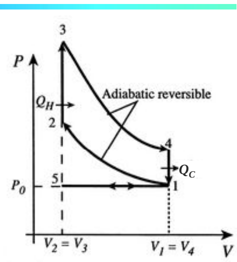
$r = \frac{V_{max}}{V_{min}}$ compression ratio

Compression:

$$T_2 V_2^{\gamma-1} = T_1 V_1^{\gamma-1} = T_1 (r V_2)^{\gamma-1} \qquad T_2 = T_1 r^{\gamma-1}$$

Expansion:

$$T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1} = T_4 (r V_3)^{\gamma-1} \qquad T_3 = T_4 r^{\gamma-1}$$



Efficiency of the Otto cycle

$$Q_H = nC_V (T_3 - T_2) \quad > 0$$
$$Q_C = nC_V (T_1 - T_4) \quad < 0$$

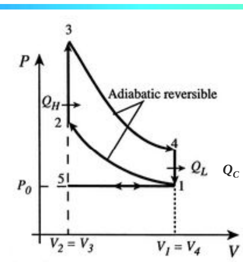
$$e = \frac{W}{Q_H} = \frac{Q_H + Q_C}{Q_H}$$

$$= \frac{T_3 - T_2 + T_1 - T_4}{T_3 - T_2}$$

$T_2 = T_1 r^{\gamma-1}$
$T_3 = T_4 r^{\gamma-1}$

$$= \frac{T_4 r^{\gamma-1} - T_1 r^{\gamma-1} + T_1 - T_4}{T_4 r^{\gamma-1} - T_1 r^{\gamma-1}}$$

$$= \frac{(T_4 - T_1)(r^{\gamma-1} - 1)}{(T_4 - T_1) r^{\gamma-1}}$$



$$\boxed{e = 1 - \frac{1}{r^{\gamma-1}}}$$

In-class example: Otto's engine efficiency

Two idealized Otto cycles have a compression ratio of 5 and 10, respectively. What is the ratio of their efficiencies? Take the gas mixture to be a diatomic gas.

$$\frac{e(r=10)}{e(r=5)} = ?$$

A. 1.27

B. 1.33

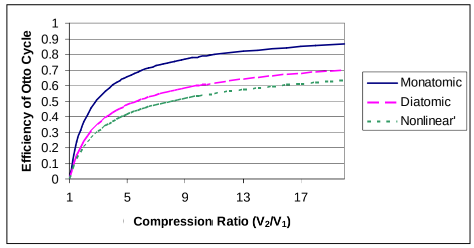
C. 1.50

D. 1.67

E. 2.00

Diatomic gas: $\gamma = \frac{C_P}{C_V} = \frac{\frac{7}{2}R}{\frac{5}{2}R} = \frac{7}{5} = 1.4$

$$\frac{e(r=10)}{e(r=5)} = \frac{1-\frac{1}{10^{1.4-1}}}{1-\frac{1}{5^{1.4-1}}} = 1.27$$



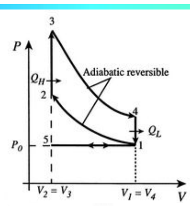
Why not simply use a higher compression ratio?

- V_2 big → huge, heavy engine
- V_1 small → temp. gets too high → premature ignition → need to use octane in gas to raise combustion temperature

Real four-stroke engine

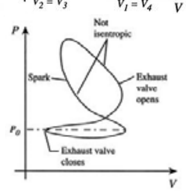
The Otto cycle is an idealization:
- assumes ideal gas
- neglects friction, turbulence, loss of heat to walls

For r = 8 and γ = 1.4 (air), e = 0.56



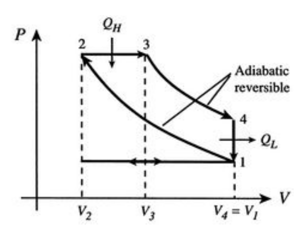
Realistic cycle of 4-stroke engine

e ~ 0.3

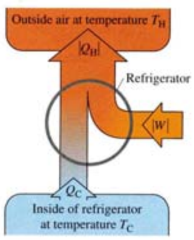


Diesel engine

- Intake and compression happen without fuel.
- Fuel is injected after compression, and keeps pressure constant.
- Compression rate r is 15-20
  - → Larger temperatures
  - → Fuel ignites spontaneously
- Ideal efficiency e ~ 0.65-0.70



Refrigerators



- Absorb heat from cold reservoir ($Q_C > 0$)
- Work done on engine ($W < 0$)
- Dump heat into hot reservoir ($Q_H > 0$)

Energy balance:

$$W = Q_H + Q_C$$
$$|W| = |Q_H| - |Q_C|$$

(We want as much Q_C while paying for the smallest possible W.)

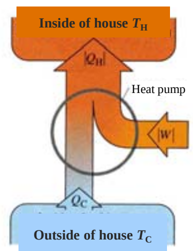
Coefficient of performance (refrigerator)

$$K = \frac{Q_C}{|W|}$$

$0 < K < \infty$

Heat pumps

A very efficient way to warm a house: bring heat from the colder outside.



Same energy diagram as refrigerator

This time we are interested in Q_H:

Coefficient of performance (heat pump)

$$K = \frac{|Q_H|}{|W|}$$

$1 < K < \infty$