Lecture 35

Heat transfer.

Mechanisms of heat transfer

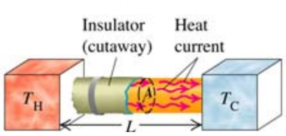
Conduction: Two surfaces in contact, or within a body.

Convection: Particles move between regions of different temperature and carry energy

Radiation: Energy carried by electromagnetic waves. Can happen through vacuum!

Conduction

Metal bar of length L and cross-sectional area A between two sources at T_{HOT} and T_{COLD}.



$$H = \frac{dQ}{dt} = kA\frac{T_H - T_C}{L}$$

k = thermal conductivity

This formula "makes sense":

What transfers energy are the collisions between faster and slower molecules, and the number of collisions increases with cross-section area, A.

Also, the rate of transfer of energy decreases with length.

Thermal conductivity is the efficiency of collisions to exchange energy

Thermal conductors and insulators

Of course, there are no perfect thermal conductors or insulators.

Conductors (high k)	Metals	50-500 W/(m K)
Insulators (low k)	Wood	0.12-0.04 W/(m K)
	Air (still)	0.024 W/(m K)
	Styrofoam	0.01 W/(m K)

How low can you go?????.... "Aerogel": 8×10^{-5} W/(m K)

What's aerogel....

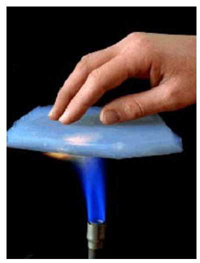
- a man-made substance formed by specially drying a wet silica gel, resulting in a solid mesh of microscopic strands.
- used on space missions to catch comet dust
- the lightest material know to man, according to the Guiness Book of World Records -- really, really light. It is ~98% porous, and yet it is quite rigid...

DEMO: Water is also a poor conductor

Aerogel

2.5 kg brick

2 g aerogel



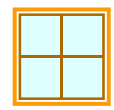
$k = 8 \times 10^{-5}$ W/m-K

Example: Window

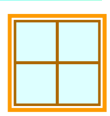
If it's 22°C inside and 0°C outside, what is the heat flow through a glass window of area 0.3 m^2 and thickness 0.5 cm?

The thermal conductivity of <u>glass</u> is about 1 W/m-K.

$$H = \frac{kA}{L}\Delta T = \left(1\frac{\text{W}}{\text{mK}}\right)\left(\frac{0.3\text{m}^2}{5\times10^{-3}\text{m}}\right)(22\text{K}) = 1320 \text{ W}$$



Example: Double pane



How much heat is lost (per second) through a double-pane version of that window, with a 0.5-cm air gap? The thermal conductivity of <u>air</u> is about 0.03 W/(m K).

We can pretty safely ignore the glass, which has a much higher conductivity than air. H is limited by conduction across the air gap.

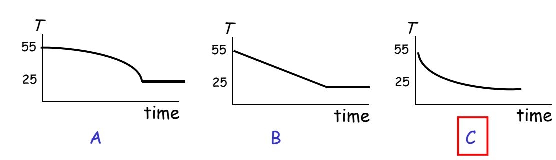
The heat current is limited by the air gap.

$$H = \left(0.03\frac{\text{W}}{\text{mK}}\right)\left(\frac{0.3\text{m}^2}{5\times10^{-3}\text{m}}\right)(22K) = 39.6 \text{ W} \quad (\ll 1320 \text{ W})$$

Note: larger air gaps don't always work better because convection currents swirl the hot and cold air around.

ACT: Hot cup

A closed container that is not a good heat insulator contains water at 55°C. It is in a room where the temperature is 25°C. Which of the following graphs shows the temperature of the water vs. time?



Consider hot cup of water, T_H, sitting in cold room T_C

- assume room is large enough so that T_C is fixed
- heat transferred from water to room at rate dQ/dt

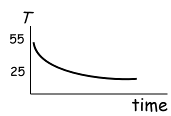
$$\frac{dQ}{dt} = kA\frac{T_H - T_C}{L}$$

A = surface area of cup
L = thickness of cup walls

What happens next ?

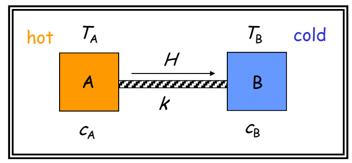
⇒ water has lost energy
⇒ T_H is reduced, closer to T_C
⇒ lower rate of heat transfer dQ/dt
⇒ This lowers the rate at which T_H is reduced

To solve find the temperature as a function of time we need a differential equation…



How long does heat conduction take?

General problem:



Isolated system: Heat released by A = Heat absorbed by B

Heat exchange through bar with thermal conductivity k

How long until thermal equilibrium?

To simplify, let us assume that one of the objects (A) is really large (ie, constant temperature).

The temperature in B changes. Every change in temperature is caused by some heat being transferred:

$$dQ = m_B c_B dT$$

At time t, the temperature of B is T. The flow rate:

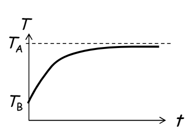
$$\frac{dQ}{dt} = \frac{kA}{L}(T_A - T)$$

$$\frac{dQ}{dt} = \frac{dQ}{dT}\frac{dT}{dt} = m_B c_B \frac{dT}{dt}$$

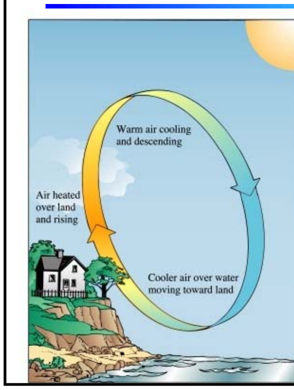
$$m_B c_B \frac{dT}{dt} = \frac{kA}{L}(T_A - T)$$

$$\frac{dT}{T_A - T} = \frac{1}{\tau}dt \qquad \tau = \frac{m_B c_B L}{kA}$$

$$\ln\frac{T_A - T}{T_A - T_B} = -\frac{t}{\tau} \qquad T - T_A = (T_B - T_A)e^{-\frac{t}{\tau}}$$



Convection



Particles move between regions of different temperature.

Mathematically… waaaay beyond 221

Example: Wind chill

Radiation

DEMO: Lighting a match

All pieces of matter emit electromagnetic radiation

- typically long-wavelength, infrared
- For hot bodies, radiation is in visible wavelengths e.g. light-bulbs

Rate of radiation of heat for a body a temperature *T*:

Stefan-Boltzmann Law $$\frac{dQ}{dt} = -Ae\sigma T^4$$

A : area of surface of body
e : emissivity of surface, empirical (e.g e = 0.3 for Cu)
σ: Stephan-Boltzmann constant
T in Kelvins

$$\sigma = 5.67 \times 10^{-8} \ \frac{\text{J}}{\text{m}^2 \cdot \text{s} \cdot \text{K}^4}$$

Radiation is in all wavelengths. Depending on *T*, most heavily in one wavelength interval than other. This is NOT described by this law

ACT: Radiating book

A book at room temperature in this room radiates energy. Yet the temperature of the book does not go down: why not?

A. Radiation stops when book is at room temperature
B. Heat is also absorbed by the book as radiation from the room
C. There must be a heat-source in the book

$$\frac{dQ_{radiated}}{dt} = -A_{book} e_{book} \sigma (T_{book})^4$$

$$\frac{dQ_{absorbed}}{dt} = +A_{book} e_{book} \sigma (T_{room})^4$$

if $T_{room} = T_{book} \quad \Rightarrow \quad \frac{dQ_{radiated}}{dt} + \frac{dQ_{absorbed}}{dt} = 0 \quad \Rightarrow$ temperature unchanged

Emission and absorption

The term radiation refers to emission ($dQ/dt < 0$) and absorption ($dQ/dt < 0$) of energy:

$$\left.\frac{dQ}{dt}\right|_{net} = -Ae\sigma\left(T^4 - T^4_{environment}\right)$$

Emissivity:

e = 1 black body: ideal emitter and ideal absorber (absorbs all the radiation that strikes it)
eg. Sun

e = 0 ideal reflector (absorbs no radiation and does not emit either)
eg. Mirrored lining inside thermos bottles

In-class example: Sitting inside a fridge

You are sitting in your shorts inside a cold (0°C) room. At what rate are you transferring heat to the room by radiation? (Take the worst case scenario with e = 1 and A = 2 m^2)

A. 0
B. 2.4×10^{-4} W
C. 404 W
D. 8.9×10^4 W
E. None of the above

Flow from body to room

$$\left.\frac{dQ}{dt}\right|_{net} = Ae\sigma\left(T^4 - T^4_{environment}\right)$$

$$= \left(2 \text{ m}^2\right)\left(5.67 \times 10^{-8} \ \frac{\text{J}}{\text{m}^2 \cdot \text{s} \cdot \text{K}^4}\right)\left(309^4 - 273^4\right)\text{K}^4$$

$$= 404 \text{ W}$$

Note: In just 10 s, this is a loss of
4000 J ~ 1000 cal = 1 Cal (food calorie).
We burn food to stay warm!