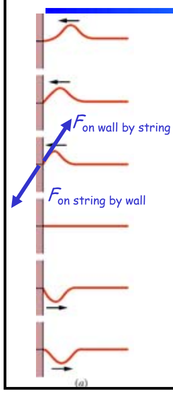
Lecture 32

Standing waves. Doppler effect.

Reflected waves: fixed end.

DEMO: Reflection

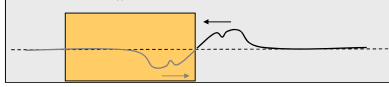


A pulse travels through a rope towards the end that is tied to a hook in the wall (ie, *fixed end*)

The force by the wall always pulls in the direction opposite to the pulse.

The pulse is inverted (simply because of Newton's 3rd law!)

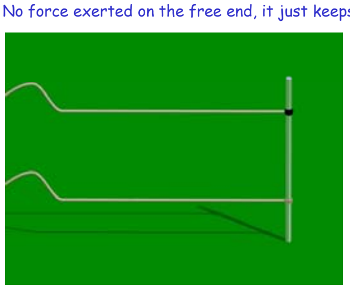
Another way (more mathematical): Consider one wave going into the wall and another coming out of the wall. The superposition must give 0 at the wall. Virtual wave must be inverted:



Reflected waves: free end.

A pulse travels through a rope towards the end that is tied to a ring that can slide up and down without friction along a vertical pole (ie, *free end*)

No force exerted on the free end, it just keeps going



Fixed boundary condition

Free boundary condition

Standing waves

A wave traveling along the $+x$ direction is reflected at a fixed point. What is the result of the its superposition with the reflected wave?

$y_1(x,t) = A\cos(kx-\omega t)$ $\quad y_2(x,t) = -A\cos(kx+\omega t)$

$$y(x,t) = A\left[\cos(kx-\omega t) - \cos(kx+\omega t)\right]$$

$$\cos(a) - \cos(b) = 2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\downarrow$$

$$\boxed{y(x,t) = 2A\sin(kx)\sin(\omega t)}$$ Standing wave

$k = \frac{2\pi}{\lambda}$

If $kx = 0, \pi, 2\pi ...$ $\Leftrightarrow$ $\boxed{x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, ...}$ $\quad y(x,t) = 0$ No motion for these points (nodes)

If $kx = \frac{\pi}{2}, \frac{3\pi}{2} ...$ $\Leftrightarrow$ $\boxed{x = \frac{\lambda}{4}, \frac{3\lambda}{4}, ...}$ $\quad y(x,t) = \pm 2A\sin(\omega t)$

These points oscillate with the maximum possible amplitude (antinodes)

Standing waves and boundary conditions

We obtained $y(x,t) = 2A\sin(kx)\sin(\omega t)$

Nodes $x = 0, \frac{\lambda}{2}, \lambda, \ldots$

Antinodes $x = \frac{\lambda}{4}, \frac{3\lambda}{4} \ldots$

We need fixed ends to be nodes and free ends to be antinodes!

↓

Strong restriction on the waves that can "survive" with a given set of boundary conditions.

Standing waves between two fixed ends

Normal modes

DEMO: Normal modes on string

Which standing waves can I have for a string of length L fixed at both ends?

I need nodes at $x = 0$ and $x = L$

Nodes $x = 0, \frac{\lambda}{2}, \lambda, \ldots$

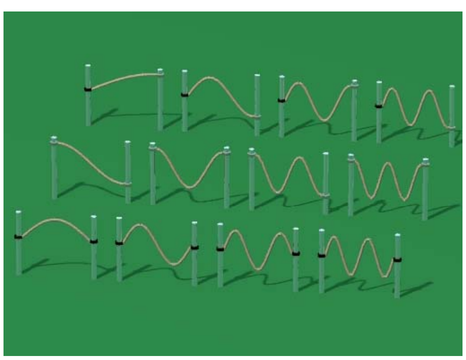
$$L = \frac{\lambda}{2}, \lambda, \ldots = n\frac{\lambda}{2} \quad \text{for } n = 1,2,\ldots$$

$$\boxed{\lambda_n = \frac{2L}{n} \quad \text{for } n = 1,2,\ldots}$$

Allowed standing waves (normal modes) between two fixed ends

Mode n = n-th harmonic

Ruben's tube



1 fixed, 1 free

2 free ends

2 fixed ends

$\lambda_1 = 2L$ First harmonic

$\lambda_2 = L$ Second harmonic…

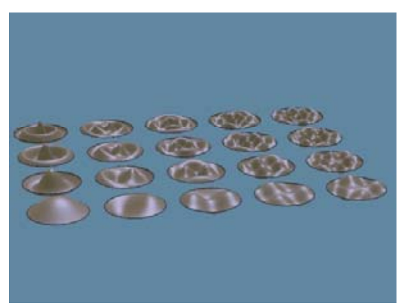
$\lambda_3 = \frac{2L}{3}$

$\lambda_4 = \frac{L}{2}$

Normal modes for fixed ends (lower row)

Normal modes 2D

DEMO: Normal modes square surface



For circular fixed boundary

Resonance

DEMO: Resonant slabs

To produce a wave, we need to apply an external force (driving force). This driving force can be periodic with frequency f_D.

The amplitude of the perturbation is maximum when the frequency of the driving force is equal to one of the natural (or harmonic, or normal) frequencies of the system.

Examples:

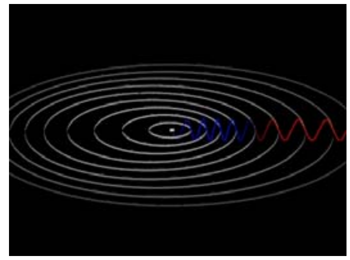
Pendulum: resonance occurs when $f_D = 2\pi\sqrt{\frac{g}{L}}$ (L = length of string)

(A pendulum has only one normal frequency)

String fixed at both ends: when $f_D = f_n = \sqrt{\frac{F}{\mu}}\frac{n}{2L}$ for $n = 1,2,...$

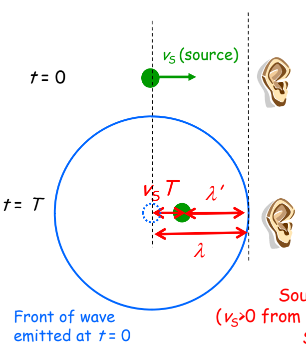
Doppler Shift

DEMO: Whistle



Even better: http://www.lon-capa.org/~mmp/applist/doppler/d.htm

Doppler math: moving source



- Speed of sound v is constant.
- Source emits λ
- Listener (ear) perceives λ'

$$\lambda' = \lambda - v_s T$$

$$\frac{v}{f'} = \frac{v}{f} - \frac{v_s}{f}$$

$$f' = f\frac{v}{v - v_s}$$

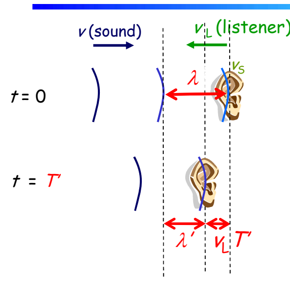
Source moving with v_S

(v_S>0 from listener to source)

Stationary listener

$$f_L = f_S\frac{v}{v + v_s}$$

Doppler math: moving listener



$$\lambda' = \lambda - v_L T'$$

$$\frac{v}{f'} = \frac{v}{f} - \frac{v_L}{f'}$$

$$f' = f\frac{v + v_L}{v}$$

Stationary source.

Source moving with v_L

(v_L>0 from listener to source)

$$f_L = f_S\frac{v + v_L}{v}$$

Moving source and moving listener

$$f_L = f_S \frac{v + v_L}{v + v_S}$$

v_L, $v_S > 0$ in direction from listener to source ($v > 0$ always)

To get signs correct
1) sketch the situation, including a few wavefronts
2) decide whether observed wavelength or period will be shorter or longer
3) use this to guide whether frequency increases, decreases
4) keep in mind speed of sound does not depend on what the source or observer is doing

ACT: Doppler

A train is approaching you as you stand on a platform at a railway station. As the train approaches, it slows down. All the while, the engineer is sounding the horn at a constant frequency of 500 Hz.

1. Heard frequency is greater than 500 Hz and increases as train slows down
2. Heard frequency is greater than 500 Hz and decreases as train slows down
3. Heard frequency is less than 500 Hz and increases as train slows down
4. Heard frequency is less than 500 Hz and decreases as train slows down

Source approaching listener: wavefronts are squeezed together: $\lambda\downarrow$ $f\uparrow$

Effect must be getting smaller (back to source frequency): f decreases

In-class example: Doppler

A source of sound has a characteristic frequency f. The speed of sound is v. Consider the following four scenarios:

1. Static source, $v_{observer}$ = $v/2$ toward source
2. Static source, $v_{observer}$ = $v/2$ away from source
3. Static observer, v_{source} = $v/2$ toward observer
4. Static observer, v_{source} = $v/2$ away from observer

Order f_1, f_2, f_3, f_4 from lowest to highest.

A. $f_1 = f_2 = f_3 = f_4$
B. $f_2 = f_4$, $f_1 = f_3$
C. f_1 , f_2 , f_3 , f_4
D. f_2 , f_4 , f_1 , f_3
E. f_4 , f_3 , f_2 , f_1

A source of sound has a characteristic frequency f. The speed of sound is v. Consider the following four scenarios:

1. Static source, $v_{observer}$ = $v/2$ toward source
2. Static source, $v_{observer}$ = $v/2$ away from source
3. Static observer, v_{source} = $v/2$ toward observer
4. Static observer, v_{source} = $v/2$ away from observer

Order f_1, f_2, f_3, f_4 from lowest to highest.

A. $f_1 = f_2 = f_3 = f_4$
B. $f_2 = f_4$, $f_1 = f_3$
C. f_1 , f_2 , f_3 , f_4
D. f_2 , f_4 , f_1 , f_3
E. f_4 , f_3 , f_2 , f_1

$$f_1 = f\frac{v + \frac{v}{2}}{v} = 1.5f \qquad f_2 = f\frac{v - \frac{v}{2}}{v} = 0.5f$$

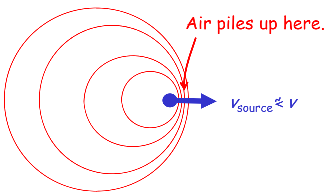
$$f_3 = f\frac{v}{v - \frac{v}{2}} = 2f \qquad f_4 = f\frac{v}{v + \frac{v}{2}} = 0.67f$$

It is NOT option B: 2 and 4 (or 1 and 3) are not equivalent.
You need to think about the motion relative to air, too.

Shock waves

What if the source (a plane, for instance) is moving almost at the speed of sound?

http://www.lon-capa.org/~mmp/applist/doppler/d.htm



High pressure and density in front of plane

↓

Large aerodynamic drag (plane pushes on air, air pushes back)

↓

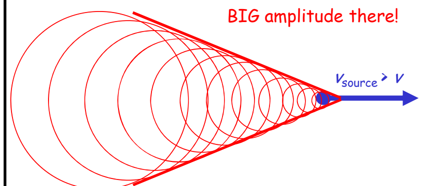
"Sound barrier"

Supersonic speeds

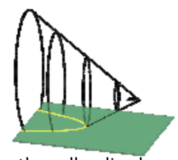
And what if $v_{source} > v$? http://www.lon-capa.org/~mmp/applist/doppler/d.htm

Points of constructive interference are along the red lines (sides of a cone)

BIG amplitude there!

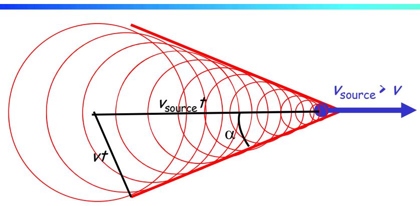


When this cone touches the ground…



…a person on the yellow line hears a <u>very</u> loud sound (sonic boom)

Mach number



$$\sin\alpha = \frac{vt}{v_{source}t} = \frac{v}{v_{source}}$$

$$\frac{v_s}{v} = \text{Mach number}$$

Sonic boom $2\alpha \sim 130°$ Mach ~ 1.1