Lecture 31

Energy. Intensity.
Interference. Beats.

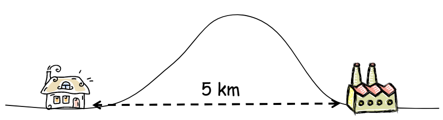
ACT: A sixth sense?

A large ammunition factory and a town are separated by a rocky hill, at a horizontal distance of about 5 km. An accident produces a <u>huge</u> explosion in the middle of the night. What do the town folks experience?

A. First the room shakes, and then they hear an explosion.

B. First they hear an explosion, and then the room shakes.

C. They hear an explosion and the room shakes at the same time.



Time for sound wave to reach the town:

Through hill (granite):

$$t = \frac{x}{v} = \frac{5000\text{ m}}{6000\text{ m/s}} = 0.8\text{ s}$$

$$\left(v = \sqrt{\frac{Y_{granite}}{\rho_{granite}}} = 6000\text{ m/s} \right)$$

Through air:

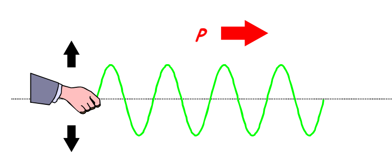
$$t = \frac{x}{v} = \frac{5000\text{ m}}{343\text{ m/s}} = 14.6\text{ s}$$

14 seconds later!

This happened in California during WWII. Most people woke up (distressed…) to the light quake and then heard the explosion. Many attributed this to a "sixth sense" that had warned them of the imminent disaster. The "sixth sense" was just the laws of wave propagation…

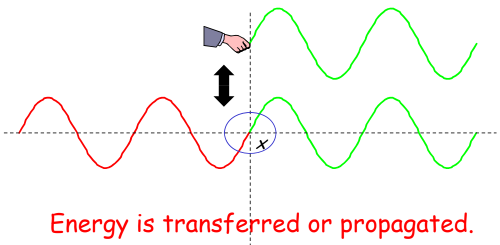
Wave energy

- Work is clearly being done: $\mathbf{F}\cdot d\mathbf{r} > 0$ as hand moves up and down.
- This energy must be moving away from your hand (to the right) since the kinetic energy (motion) of the end of the string grabbed by the hand stays the same.



Transfer of energy

The string to the left of x does work on the string to the right of x, just as your hand did:



Energy is transferred or propagated.

Power

Energy for a particle in SHM (attached to a spring k)

$$E = \frac{1}{2}kA^2 \propto \omega^2 A^2 \qquad \left(\omega^2 = \frac{k}{m}\right)$$

This energy propagates at speed v.

$\Rightarrow$ the average energy per unit time that flows in the direction of propagation should be proportional to v

$$\langle P \rangle \propto v\omega^2 A^2$$ Average power

For harmonic waves (see appendix):

$$\langle P \rangle = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2 = \frac{1}{2}\frac{v}{\mu}\omega^2 A^2$$ Average power for harmonic waves

Intensity

$$I = \frac{\langle P \rangle}{\text{area}}$$

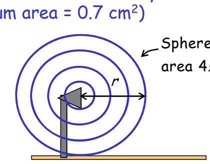
Average power (over time) in wave

Area of the surface where this power is distributed

Example: A siren emits a sound of power 2W at 100 m from you.
How much power reaches your ear (eardrum area = 0.7 cm^2)

Intensity at distance r from source:

$$I_R = \frac{P_{\text{at source}}}{4\pi r^2} = \frac{2\text{ W}}{4\pi(100\text{ m})^2} = 1.6\times10^{-5}\text{ W/m}^2$$



Power absorbed by eardrum:

$$P_{\text{eardrum}} = I_R \times (\text{area of eardrum}) = (1.6\times10^{-5}\text{ W/m}^2)(0.7\times10^{-4}\text{m}^2) = 1.1\text{ nW}$$

ACT: Waves and friction

Consider a traveling wave that loses energy to friction
If it loses half the energy and its shape stays the same, what is amplitude of the wave ?

A. amplitude decreases by ½
B. amplitude decreases by 1/√(2)
C. amplitude decreases by ¼

Power is proportional the amplitude squared A^2.

Distance and amplitude

At distance r from the source, the power is $P_r \propto I_r \propto \frac{1}{r^2}$

We also know that $P \propto (\text{Amplitude})^2$

Amplitude decreases as $\frac{1}{r}$

Sound intensity level

$$\beta = 10\log\frac{I}{I_0} \quad \text{with } I_0 = 10^{-12}\ \text{W/m}^2$$

Units: decibels

Threshold of human hearing: 10^{-12} W/m^2 $\rightarrow$ $\beta = 0$

Normal conversation: 10^{-6} W/m^2 $\rightarrow$ $\beta = 65$ decibels

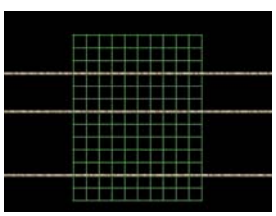
Threshold of pain: 1 W/m^2 $\rightarrow$ $\beta = 120$ decibels

Twice the decibels does NOT feel twice as loud!

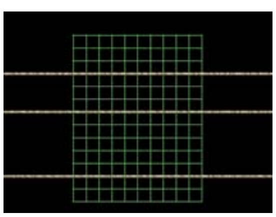
Interference, superposition

Q: What happens when two waves "collide?"

A: They ADD together! We say the waves are ***superposed***.



Constructive

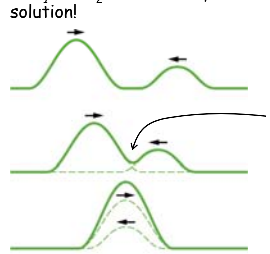


Destructive

Why superposition works

The wave equation is linear: It has no terms where variables are squared.

If f_1 and f_2 are solution, then $Bf_1 + Cf_2$ is also a solution!



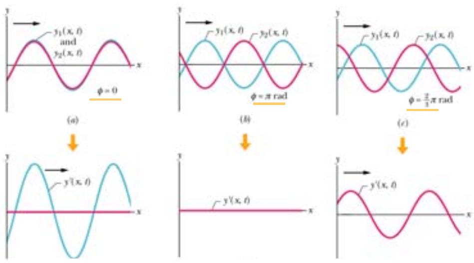
These points are now displaced by both waves

Superposition of two identical harmonic waves out of phase

Two identical waves out of phase:

$y_1(x,t) = A\cos(kx - \omega t)$ $\quad$ $y_2(x,t) = A\cos(kx - \omega t + \boxed{\phi})$

Wave 2 is little ahead or behind wave 1



constructive $\quad$ destructive $\quad$ intermediate

Superposition of two identical harmonic waves out of phase: the math

$y_1(x,t) = A\cos(kx - \omega t) \qquad y_2(x,t) = A\cos(kx - \omega t + \phi)$

$$y(x,t) = y_1(x,t) + y_2(x,t) = A\cos(kx - \omega t) + A\cos(kx - \omega t + \phi)$$
$$= A\left[\cos(kx - \omega t) + \cos(kx - \omega t + \phi)\right]$$

$$\cos(a) + \cos(b) = 2\cos\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)$$

$$\boxed{y(x,t) = 2A\cos\left(\frac{\phi}{2}\right)\cos\left(kx - \omega t + \frac{\phi}{2}\right)}$$

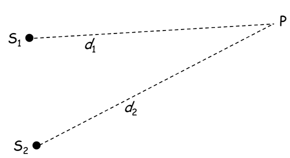
When $\phi = \pi$ interference is completely destructive

$\phi = 0$ interference is completely constructive

It's all about the phase difference

Interference for sound (3D)

Let S_1, S_2 be two sources that emit spherical sound waves in phase.



At point P:

$s_1(\vec{r},t) = s_{1max}\cos(kd_1 - \omega t)$

$s_2(\vec{r},t) = s_{2max}\cos(kd_2 - \omega t)$

$\boxed{\text{Phase difference} = k(d_2 - d_1)}$

This is what matters…

Destructive interference

$k(d_2 - d_1) = n_{odd}\pi \qquad d_2 - d_1 = n_{odd}\frac{\lambda}{2}$

If both waves have the same amplitude (equal distance to sources), these points can even have zero intensity!

+ amplitude $\quad$ - amplitude

Interference in real life?

Your stereo equipment does not seem to produce these minimum intensity spots…

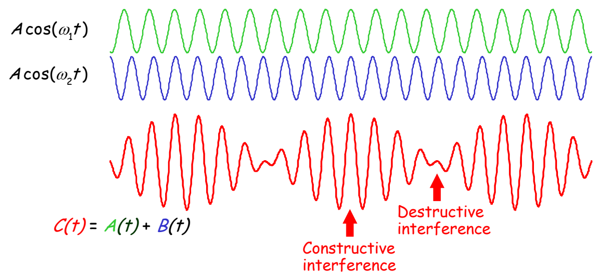
- many frequencies at the same time
- multiple reflections on walls, ceiling, furniture…

DEMO: Interference

Beats

Consider two harmonic waves meeting at $x = 0$. Same amplitudes, but $\omega_2 = 1.15\ \omega_1$.

The displacement versus time for each is shown below:

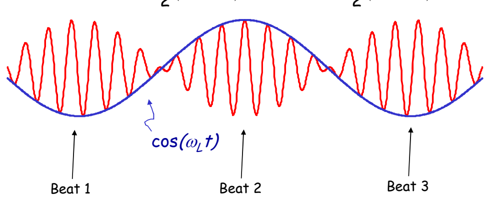


Beats (math)

DEMO: Beats

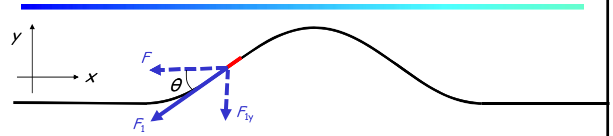
$$A\cos(\omega_1 t) + A\cos(\omega_2 t) = 2A\cos(\omega_L t)\cos(\omega_H t)$$

where $\omega_L = \frac{1}{2}(\omega_1 - \omega_2)$ and $\omega_H = \frac{1}{2}(\omega_1 + \omega_2)$



Note: What you actually hear (beats) has frequency $f_{beat} = 2f_L = |f_1 - f_2|$

Appendix: Power



What is the work done on the red segment by the string to its left?

The red segment moves in the y direction with velocity $\frac{\partial y}{\partial t}$

and is subject to a force whose y component is $F_{1y} = -|F_{1x}|\tan\theta = -F\frac{\partial y}{\partial x}$

with $|F_{1x}| = |F|$ (tension in the string)

Power (how does energy move along the wave)

$$P = \vec{F}_1 \cdot \vec{v}_1 = F_{1y} v_{1y} \qquad P = -F\frac{\partial y}{\partial x}\frac{\partial y}{\partial t}$$

Power for harmonic waves

For a harmonic wave, $y(x,t) = A\cos(kx - \omega t)$

$$\frac{\partial y}{\partial x} = -kA\sin(kx - \omega t) \qquad \text{and} \qquad \frac{\partial y}{\partial t} = \omega A\sin(kx - \omega t)$$

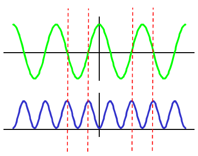
$$P = -F\frac{\partial y}{\partial x}\frac{\partial y}{\partial t} = Fk\omega A^2\sin^2(kx - \omega t)$$

$$k = \frac{\omega}{v} = \frac{\omega}{\sqrt{\frac{F}{\mu}}}$$

$$P(x,t) = \sqrt{\mu F}\,\omega^2 A^2 \sin^2(kx - \omega t)$$

Power (energy flow along x direction)

NB: Always positive, as expected



$\cos(kx - \omega t)$

$\sin^2(kx - \omega t)$

Maximum power where vertical velocity is largest ($y = 0$)

Average power for harmonic waves

Average over time: $\langle P \rangle = \sqrt{\mu F}\,\omega^2 A^2 \langle \sin^2(kx - \omega t) \rangle$

$$\langle \sin^2\alpha \rangle = \frac{\int_0^{2\pi} \sin^2\alpha \, d\alpha}{2\pi} = \frac{1}{2}$$

$$\langle P \rangle = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2 = \frac{1}{2}\frac{v}{\mu}\omega^2 A^2$$

Average power for harmonic waves

It is generally true (for other wave shapes) that wave power is proportional to the speed of the wave v and its amplitude squared A^2.