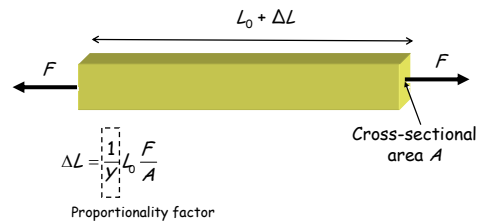


Lecture 26

Stress, strain and elasticity

Linear deformation of a solid



Young's modulus

$$Y = \frac{F/A}{\Delta L/L_0}$$

stress

strain (deformation)

Elastic moduli

Hooke's law: $\frac{\text{stress}}{\text{strain}} = \text{constant (elastic modulus)}$

This works for small strains.

Remember springs? $\frac{F}{\Delta x} = k$ (We absorbed A and L_0 into k)

Unit of stress:

SI: Pascal 1 Pa = 1 N/m²

US: psi (pounds per square inch)

Young's modulus

$$Y = \frac{F/A}{\Delta L/L_0} \quad \text{Measure of stiffness}$$

Material	Young's modulus (GPa)
Rubber (small strain)	0.01-0.1
wood	1-10
bone	9-16
concrete	20
steel	200

In-class example: Young's modulus

When a tensile stress of 5×10^6 Pa is applied to the ends of a bar, it is desired that the strain be about 5×10^{-4} . The most appropriate material to use for this bar would be:

- A. Rubber ($Y \sim 3 \times 10^7$ Pa)
- B. Wood ($Y \sim 1 \times 10^{10}$ Pa)
- C. Brass ($Y \sim 1 \times 10^{11}$ Pa)
- D. New material ($Y \sim 10^{12}$ Pa)
- E. None of these come close.

$$Y = \frac{\text{stress}}{\text{strain}} = \frac{5 \times 10^6 \text{ Pa}}{5 \times 10^{-4}} = 10^{10} \text{ Pa}$$

What is a fluid?

Fluids are "substances that flow" "substances that take the shape of the container"

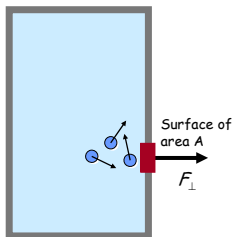
Atoms and molecules must be free to move .. No long range correlation between positions (e.g., not a crystal).

Gas or liquid... or granular materials (like sand)

Pressure in a fluid

DEMO: Plastic glass with cover

Particles are always moving → i.e., hitting surfaces → i.e., exerting (perpendicular) forces on surfaces



Pressure $p = \frac{F_{\perp}}{A}$

Does not depend on orientation of surface

Units: Pascal (Pa) = 1 N/m²
psi (pounds per square inch)
atmosphere 1 atm = 1.013 × 10⁵ Pa
bar 1 bar = 10⁵ Pa

Atmospheric pressure

The atmosphere of Earth is a fluid, so every object in air is subject to some pressure.

At the surface of the Earth, the pressure is

$$p_{\text{atm}} \sim 1.013 \times 10^5 \text{ Pa} = 1 \text{ atm}$$

Area of a hand ~ 200 cm² = 0.02 m²
 $F = p_{\text{atm}} A \sim 2000 \text{ N}$ on your hand due to air!

DEMO: Piston and weight



Pressure vs. depth

DEMO: Container with holes

Imaginary box of fluid with bases of area A and height h

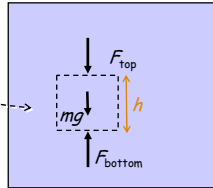
Net force must be zero!

$$F_{\text{bottom}} = F_{\text{top}} + mg$$

$$\rho_{\text{bottom/top}} = \frac{F_{\text{bottom/top}}}{A}$$

$$m = \rho Ah$$

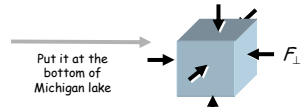
$$\rho_{\text{bottom}} = \rho_{\text{top}} + \rho gh$$



Bulk stress and strain



pressure p_0
volume V_0



pressure $p_0 + \Delta p$
volume $V_0 + \Delta V$ ($\Delta V < 0$)

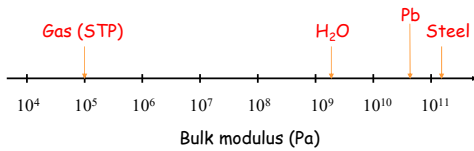
Bulk modulus $B = -\frac{\Delta p}{\Delta V / V_0}$ stress strain

Compressibility $k = \frac{1}{B}$

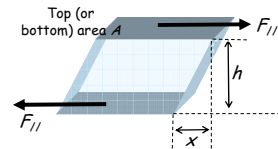
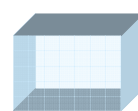
Bulk modulus

GAS: compressible, small B (density depends a lot on pressure)

LIQUID and SOLIDS: (nearly) incompressible, large B (density almost "constant")



Shear modulus

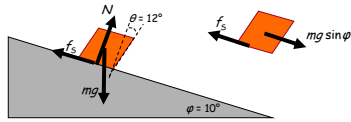


Shear modulus $S = \frac{F_{\parallel} / A}{x / h}$ stress strain

Shear angle θ
strain = $\tan \theta = \frac{x}{h}$

Example: Jello cube

A Jello cube of side $d = 4$ cm is placed on a 10° incline. It tilts to an angle of 12° . What is the shear modulus of Jello?



$$F_{\parallel} = mg \sin \phi = f_s$$

$$m = d^3 \rho_{\text{Jello}} \approx d^3 \rho_{\text{water}}$$

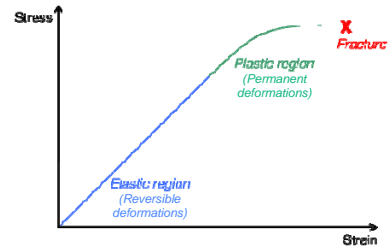
$$A = d^2$$

$$\text{strain} = \tan \theta$$

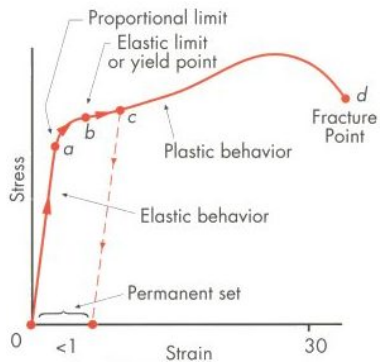
$$S = \frac{d^3 \rho_{\text{water}} g \sin \phi / \tan \theta}{d^2} = \frac{d \rho_{\text{water}} g \sin \phi}{\tan \theta}$$

$$= \frac{(4 \times 10^{-2} \text{ m})(10^3 \text{ kg/m}^3)(10 \text{ m/s}^2) \sin 10^\circ}{\tan 12^\circ} \sim 300 \text{ Pa}$$

Beyond Hooke's law



A little more realistic

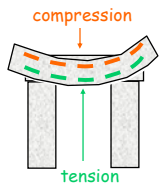


Compressive vs. tensile strength

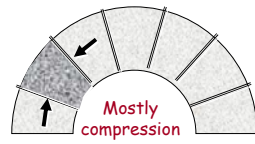
Material	Tensile strength (MPa)	Compressive strength (MPa)
steel	500	500
cast iron	170	550
concrete	2	20
marble	-	80
wood (parallel to grain)	40	35
bone	130	170

Arches

Material	Tensile strength (MPa)	Compressive strength (MPa)
concrete	2	20
marble	-	80
wood (parallel to grain)	40	35



Ok with wood, not stone



Good design for stone