Lecture 25

Statics

Equilibrium

A system is in equilibrium if no part of it is moving.

Conditions for equilibrium:

$$\sum \vec{F} = 0 \quad \textbf{and} \quad \sum \vec{\tau} = 0$$

CM is at rest (for $\sum \vec{F} = 0$)

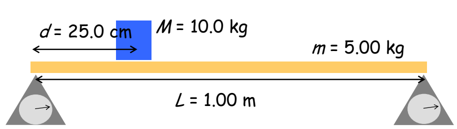
Parts are not moving about the CM. (for $\sum \vec{\tau} = 0$)

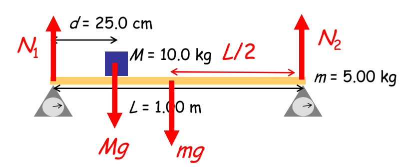
A very useful feature: The system should not rotate about ANY axis. We can take the torques about any point we like.

Example: Board on scales

A 1.00 m long board with a mass of 5.00 kg is supported at each end by a scale. A 10.0-kg mass is placed 25.0 cm from the left side. What value does each scale read?

A. 15.0 kg (L), 0 kg (R)
B. 12.5 kg (L), 2.50 kg (R)
C. 10.0 kg (L), 5.00 kg (R)
D. 7.50 kg (L), 7.50 kg (R)
E. 5.00 kg (L), 10.0 kg (R)





$$\sum \vec{F} = 0 \qquad N_1 + N_2 - Mg - mg = 0$$

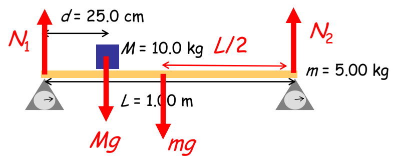
$\sum \vec{\tau} = 0$ Let us take the torque about the left end:

$$LN_2 - dMg - \frac{L}{2}mg = 0$$

2 equations, 2 unknowns

We want to solve for $\frac{N}{g}$ (scales read in kg – or lb).

DEMO:
Plank and scales



$$N_1 + N_2 - Mg - mg = 0$$
$$\Downarrow$$
$$\frac{N_1}{g} = M + m - \frac{N_2}{g}$$
$$= (5.00 + 10.0 - 5.00)\ \text{kg}$$
$$= \boxed{10.0\ \text{kg}}$$

On the left

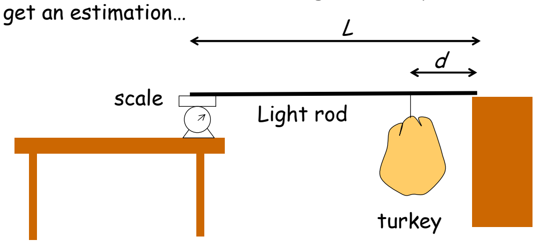
$$LN_2 - dMg - \frac{L}{2}mg = 0$$
$$\Downarrow$$
$$\frac{N_2}{g} = M\frac{d}{L} + \frac{m}{2}$$
$$= (10.0\ \text{kg})\frac{25.0\ \text{cm}}{100\ \text{cm}} + \frac{5.00\ \text{kg}}{2}$$
$$= \boxed{5.00\ \text{kg}}$$

On the right

Answer C

ACT: How to weight a turkey on a tiny kitchen scale

You bought a frozen turkey and forgot how many pounds it was. All you have is a tiny kitchen scale that can weigh a maximum of 2 lb. But here's a good trick you can use to get an estimation…



When d = 20 cm, the scale reads 2 lb exactly. If L = 1 m, how big is your turkey?

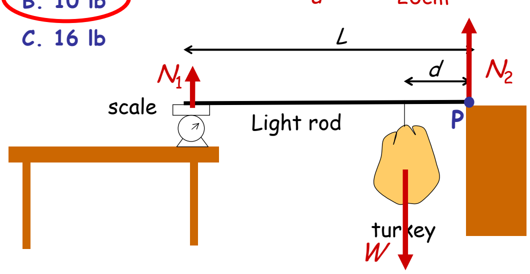
A. 8 lb
B. 10 lb
C. 16 lb

$(N_1 + N_2 - W = 0)$

$N_1 L - Wd = 0$

(torque about point P)

$$W = N_1\frac{L}{d} = 2\ \text{lb}\frac{100\text{cm}}{20\text{cm}} = 10\ \text{lb}$$



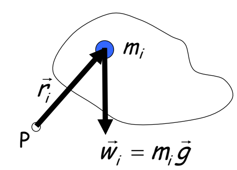
Center of mass/Center of Gravity

Torque due to gravity about an arbitrary point P:

For each m_i: $\vec{\tau}_i = \vec{r}_i \times m_i\vec{g}$

For the whole body:

$$\vec{\tau}_{\text{by gravity}} = \sum_i \vec{r}_i \times m_i\vec{g}$$
$$= \left(\sum_i m_i\vec{r}_i\right) \times \vec{g}$$
$$= M\vec{r}_{CM} \times \vec{g}$$
$$= \vec{r}_{CM} \times M\vec{g}$$

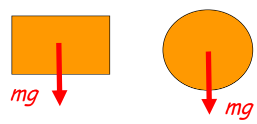


The torque due to gravity is the same as if all the mass of the object was concentrated at a point called the center of gravity.

If g is constant,

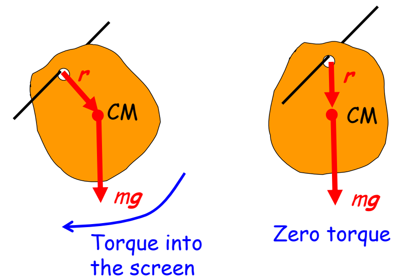
center of gravity = center of mass

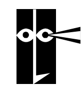
… and this is why we should draw the weight vector at the center of mass of a system.



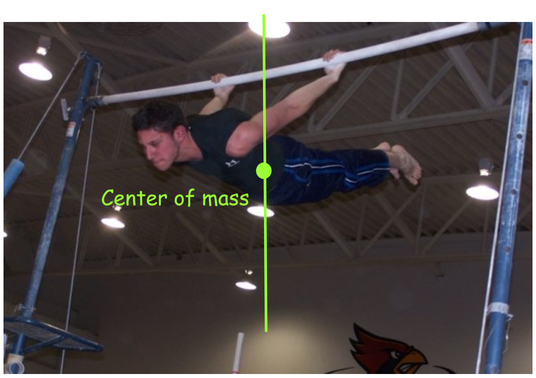
How to find the CM

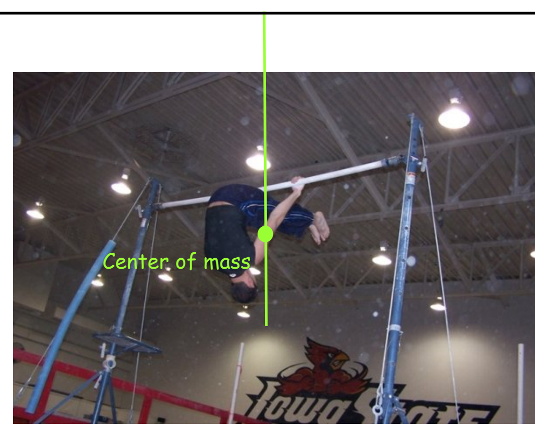
The object below is hanged from a hole drilled at point P. When is the system in equilibrium?





DEMO:
CM of Iowa



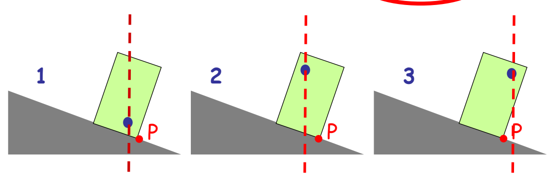


ACT: Box on incline

DEMO: Tricky disk and Tower of Pisa

Three boxes are placed on identical inclines. Friction prevents them from sliding. The boxes are not uniform and their center of mass are indicated by the blue dot in each case.
In which cases does the box tip over?

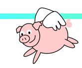
A. All **B. 2 & 3** **C. 3 only**



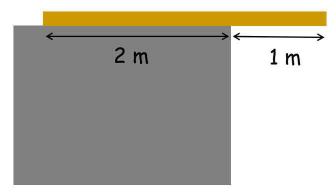
In 3, the CM is not above the base, so $\tau_{\text{net about P}} \neq 0$

(In cases 1 and 2, $\tau_N + \tau_g = 0$)

Example: Walking the plank



A 40-kg plank rests on a roof as shown. A 50-kg flying pig is about to land on the unsupported side. How far from the edge of the roof can the pig land without tipping the plank and itself over?



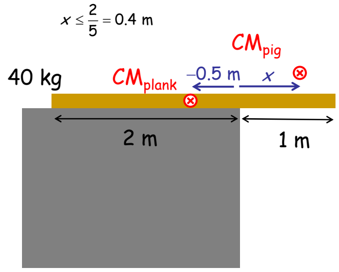
The center of mass of the plank+pig system must not be beyond the edge.

Position of the CM_{all} relative to the edge:

$$x_{CM} = \frac{(40\text{ kg})(-0.5\text{ m}) + (50\text{ kg})x}{90\text{ kg}} \leq 0$$

$$-20 + 50x \leq 0$$

$$x \leq \frac{2}{5} = 0.4\text{ m}$$

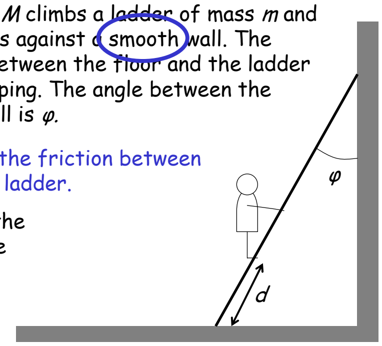


EXAMPLE: Ladder

A person of mass M climbs a ladder of mass m and length L that leans against a smooth wall. The frictional force between the floor and the ladder keeps it from slipping. The angle between the ladder and the wall is φ.

We can neglect the friction between the wall and the ladder.

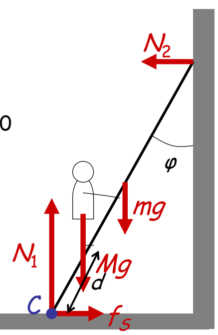
Determine how the magnitude of the frictional force depends on φ.



$$\sum F_x = 0 \qquad f_S - N_2 = 0$$
$$\sum F_y = 0 \qquad N_1 - Mg - mg = 0$$
$$\sum \tau = 0 \qquad Mgd\sin\varphi + mg\frac{L}{2}\sin\varphi - N_2 L\cos\varphi = 0$$

$$N_1 = (M+m)g$$
$$Mgd\sin\varphi + mg\frac{L}{2}\sin\varphi - f_s L\cos\varphi = 0$$

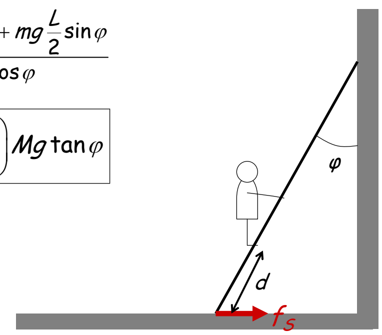
C is a good point to use as axis of rotation because then two of the forces will produce zero torque.



$$N_1 = (M+m)g$$
$$Mgd\sin\varphi + mg\frac{L}{2}\sin\varphi - f_s L\cos\varphi = 0$$

$$f_S = \frac{Mgd\sin\varphi + mg\frac{L}{2}\sin\varphi}{L\cos\varphi}$$

$$\boxed{f_S = \left(\frac{d}{L} + \frac{m}{2M}\right) Mg\tan\varphi}$$



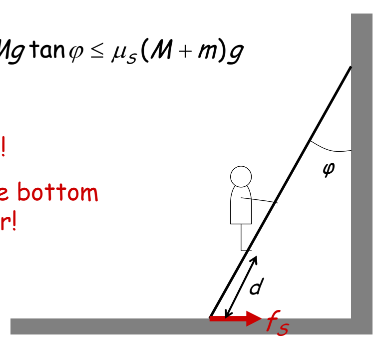
But for a given floor and ladder (a given μ_S)

$$f_{S,MAX} = \mu_S N_1 = \mu_S (M+m)g$$

$$f_S = \left(\frac{d}{L} + \frac{m}{2M}\right) Mg\tan\varphi \le \mu_S (M+m)g$$

Keep φ small!

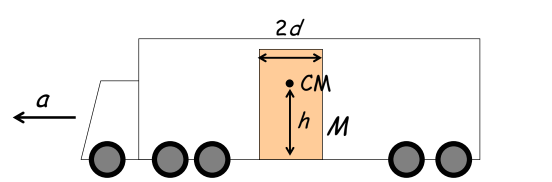
Or brace the bottom of the ladder!



EXAMPLE: Moving a fridge

A truck carries a refrigerator of mass M on a horizontal road. The center of mass of the fridge is at height h from the bed of the truck and the width of the fridge is $2d$.

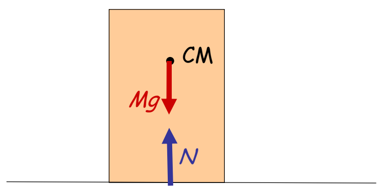
What is the maximum acceleration a_M that the truck can have without tipping the fridge? (Assume the static friction between the truck and the fridge is large enough for the fridge not to slip).



When the truck has no acceleration (or is at rest), the normal force is right below the CM.

$$\sum F_y = 0 \qquad N = Mg$$

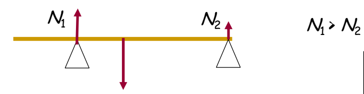
$$\sum \tau = 0 \qquad \text{(trivial)}$$



When the truck has some acceleration (< a_M) to the left, the normal force "moves" to the right to produce enough torque (about the CM) to cancel the torque due to friction (about the CM)!

"Moves" = "The pressure on the ground is redistributed"

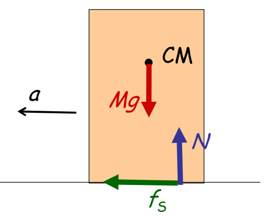
Just like in the case of a plank resting on two points:



$$\sum F_x = Ma \qquad f_s = Ma$$

$$\sum F_y = 0 \qquad N = Mg$$

$$\sum \tau = 0 \qquad \tau_f - \tau_N = 0$$

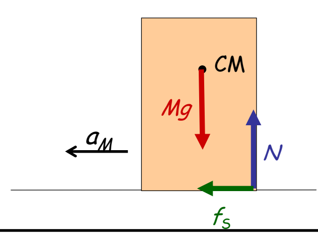


When the fridge is just starting to tip over, the normal acts on the bottom right edge of the fridge.

$$\sum F_x = Ma \qquad f_s = Ma$$

$$\sum F_y = 0 \qquad N = Mg$$

$$\sum \tau = 0 \qquad \tau_f - \tau_N = 0 \qquad f_s h - Nd = 0$$



$$f_s = Ma$$

$$N = Mg \qquad \longrightarrow \qquad Ma_M h - Mgd = 0$$

$$f_s h - Nd = 0$$

$$\boxed{a_M = \frac{d}{h} g}$$

The fridge tips over because the normal force cannot produce a torque larger than Nd.