Lecture 19

Elastic and Inelastic Collisions

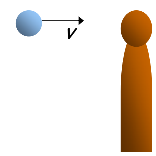
ACT: Bowling pin

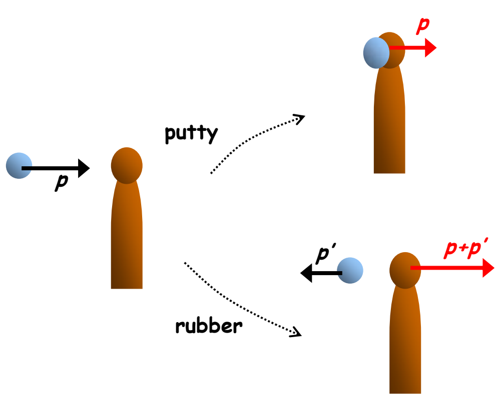
You want to knock down a large bowling pin by throwing a ball at it. You can choose between two balls of equal mass and size. One is made of rubber and bounces back when it hits the pin. The other is made of putty and sticks to the pin. Which ball do you choose?

A. The rubber ball.

B. The putty ball.

C. It makes no difference.





Collisions, explosions

These are situations where $F_{\text{internal}} >> F_{\text{external}}$

$\downarrow$

$F_{\text{external}} \sim 0$

$\downarrow$

Total linear momentum is conserved.

Or: The momentum transfer due to the internal forces is much larger than that due to external forces.

What happens to the total kinetic energy?

1. Constant $K \longrightarrow$ Elastic collisions

   When internal forces are conservative or objects are "hard".
   Examples: Elementary particles, billiard balls

2. K decreases $\longrightarrow$ Inelastic collisions

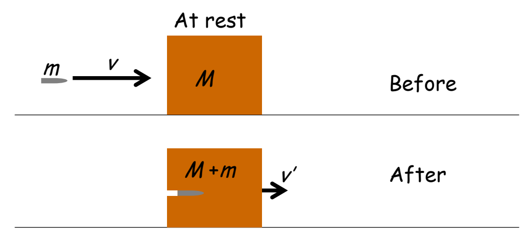
   Whenever a deformation is involved.
   Example: Most macroscopic collisions

   Special case: Perfectly inelastic collisions, when the objects stick together. Example: Pin and putty

3. K increases
   - Explosions: One body breaks into a number of parts. The explosion mechanism provides the extra energy.
   - Superelastic collisions: Some internal energy is transformed into KE because of a collision.
     Example: An excited atom hits another atom and drops to a lower state without radiation.

1D Inelastic: Shooting at a block

A block of mass M is initially at rest on a frictionless horizontal surface. A bullet of mass m is fired at the block with speed v. The bullet lodges in the block. Determine the final speed of the block (with the bullet).



$p_{\text{total, initial}} = mv + 0$

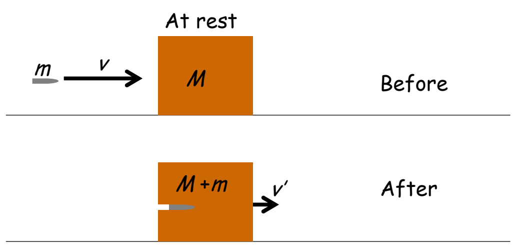
$p_{\text{total, final}} = (M + m)v'$

$\longrightarrow \quad mv = (M + m)v'$

$$\boxed{v' = \frac{m}{M+m}v}$$

$\rightarrow 0$ as $M \rightarrow \infty$ ✓

$\rightarrow v$ as $M \rightarrow 0$ ✓



Is total kinetic energy conserved?

$$KE_{\text{total, initial}} = \frac{1}{2}mv^2$$

$$KE_{\text{total, final}} = \frac{1}{2}(M+m)v'^2 = \frac{1}{2}(M+m)\left(\frac{m}{M+m}v\right)^2 = \frac{1}{2}\frac{m}{M+m}mv^2$$

Clearly $\neq$

$\Longrightarrow$ inelastic collision

$$\Delta KE_{\text{total}} = KE_{\text{total, final}} - KE_{\text{total, initial}} =$$

$$= \frac{1}{2}mv^2\left(\frac{m}{M+m} - 1\right) =$$

$$= -\frac{1}{2}\frac{M}{M+m}mv^2 < 0$$

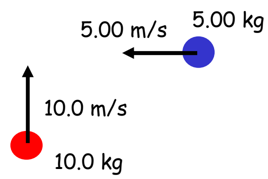
(Work done by friction to stop the bullet)

2D (Totally) Inelastic: Sticking Together

(Totally) inelastic

The two masses in the figure collide and stick together. They are moving on a horizontal, frictionless surface. What is the change in kinetic energy for this process?

- ~~A. 0 J~~
- B. -104 J
- C. -208 J
- D. -312 J
- E. -416 J



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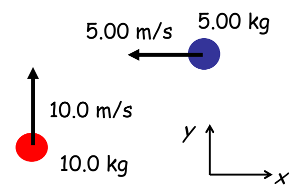


$$\vec{p}_{\text{TOTAL, BEFORE}} = \vec{p}_{\text{TOTAL, AFTER}}$$

$$(5\text{ kg})(-5\text{ m/s})\hat{i} + (10\text{ kg})(10\text{ m/s})\hat{j} = (15\text{ kg})\vec{v}$$

$$\vec{v} = \left(-1.67\hat{i} + 6.67\hat{j}\right)\text{ m/s}$$

$$\Delta KE = KE_{AFTER} - KE_{BEFORE} =$$

$$= \frac{1}{2}(15)(1.67^2 + 6.67^2) - \left[\frac{1}{2}(10)(10^2) + \frac{1}{2}(5)(5^2)\right] = -208\text{ J}$$

ACT: Big block, small block

Consider the following two collisions between two blocks of masses *m* and *M* (> *m*). In both cases, one of the blocks is initially moving with speed *v* and the other is at rest. After the collision, they move together. The final speed of the two objects is larger when:

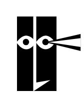
- A. The big block is initially at rest.
- B. The small block is initially at rest.
- C. The speed is the same in both cases.

Use conservation of linear momentum:

$$mv = (M+m)v_A \quad \Rightarrow \quad v_A = \frac{m}{m+M}v$$

$$Mv = (M+m)v_B \quad \Rightarrow \quad v_B = \frac{M}{m+M}v > v_A$$

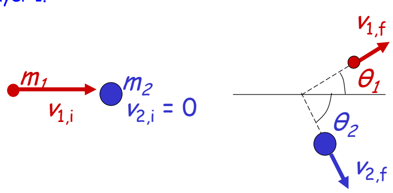
DEMO: Sticky gliders



2D Inelastic: Hockey players

A hockey player of mass m_1 = 80 kg hits another player of mass m_2 = 70 kg that is initially at rest. The final speed of the player 1 is v_{1f} = 6.0 m/s. He comes out at an angle θ_1 = 40° with its original direction. Player 2 comes out at an angle θ_2 = 65°.

Determine the final speed of player 2 and the initial speed of player 1.



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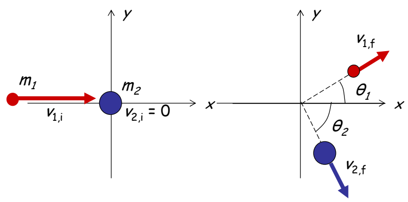
Determine the final speed of player 2 and the initial speed of player 1.

$$x: \quad m_1 v_{1,i} + 0 = m_1 v_{1,f}\cos\theta_1 + m_2 v_{2,f}\cos\theta_2$$

$$y: \quad 0 + 0 = m_1 v_{1,f}\sin\theta_1 - m_2 v_{2,f}\sin\theta_2$$



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$$x: \quad 80 v_{1,i} = 80\times 6\cos 40^\circ + 70 v_{2,f}\cos 65^\circ$$

$$y: \quad 0 = 80\times 6\sin 40^\circ - 70 v_{2,f}\sin 65^\circ$$

$$x: \quad 8 v_{1,i} = 48\cos 40^\circ + 7 v_{2,f}\cos 65^\circ$$

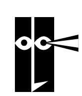
$$y: \quad 0 = 48\sin 40^\circ - 7 v_{2,f}\sin 65^\circ$$

$$y: \quad v_{2,f} = \frac{48\sin 40^\circ}{7\sin 65^\circ} = 4.9\ \text{m/s}$$

$$x: \quad v_{1,i} = \frac{48\cos 40^\circ + 7 v_{2,f}\cos 65^\circ}{8} = \frac{48\cos 40^\circ + 7(4.9)\cos 65^\circ}{8} = 6.4\ \text{m/s}$$

Explosions

Just invert final and initial states in perfectly elastic collision.

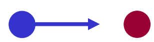


DEMO:

Propulsion/Firecracker in glider

1D Elastic: Two steel balls head-on

A steel ball with mass m_1 = 1 kg and initial speed v_0 collides head-on with another ball of mass m_2 = 2 kg that is initially at rest. What are the final speeds of the balls?



The hard way to solve this:

Head-on: 1D process.

$$v_0 + 0 = v_1 + 2v_2$$

Hard steel balls: elastic collision (KE_{total} is conserved)

$$\frac{1}{2}v_0^2 + 0 = \frac{1}{2}v_1^2 + \frac{1}{2}2v_2^2$$
$$v_0^2 = v_1^2 + 2v_2^2$$

$$v_0 = v_1 + 2v_2 \qquad v_1 = v_0 - 2v_2$$
$$v_0^2 = v_1^2 + 2v_2^2 \qquad v_0^2 = (v_0 - 2v_2)^2 + 2v_2^2$$

$$v_1 = v_0 - 2v_2$$
$$6v_2^2 - 4v_0 v_2 = 0$$

Two solutions:

$v_2 = 0 \qquad v_1 = v_0 \leftarrow$ No collision!

$v_2 = \frac{2}{3}v_0 \qquad v_1 = -\frac{1}{3}v_0$

But… there is an easier way to solve elastic collisions.

$$m_1 v_0 = m_1 v_1 + m_2 v_2 \qquad \rightarrow \qquad v_0 = v_1 + \frac{m_2}{m_1}v_2$$
$$\frac{1}{2}m_1 v_0^2 = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 \qquad\qquad v_0^2 = v_1^2 + \frac{m_2}{m_1}v_2^2$$

$$\frac{m_2}{m_1}v_2 = v_0 - v_1$$
$$\frac{m_2}{m_1}v_2^2 = v_0^2 - v_1^2$$

$$v_2 = v_0 + v_1$$

$$v_0 = -(v_1 - v_2)$$

Velocity of 1 relative to 2 before the collision

Velocity of 1 relative to 2 after the collision

So the relative velocity has the same magnitude and opposite sign before and after the collision:

$$v_0 = -(v_1 - v_2)$$

But… relative velocity is the same independently of the original frame of reference (including one in which both balls are initially moving)

The relative velocity has the same magnitude and opposite sign before and after any elastic collision between two bodies:

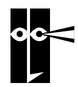
$$v_{1,i} - v_{2,i} = -(v_{1,f} - v_{2,f})$$

Let's try the problem again.

A steel ball with mass m_1 = 1 kg and initial speed v_0 collides head-on with another ball of mass m_2 = 2 kg that is initially at rest. What are the final speeds of the balls?

$$v_0 + 0 = v_1 + 2v_2 \qquad v_0 = v_1 + 2v_2 \qquad 2v_0 = 3v_2$$

$$v_0 - 0 = -(v_1 - v_2) \qquad v_0 = v_2 - v_1$$



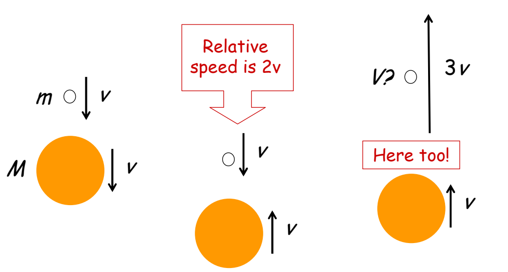
DEMO: Basketball and superball - Astroblaster

$$v_2 = \frac{2}{3}v_0 \quad ; \quad v_1 = -\frac{1}{3}v_0$$

No quadratic equations!

DEMO: Basketball and superball

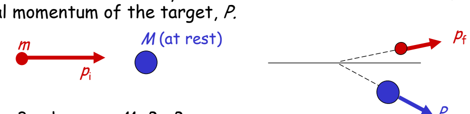
Assume all collisions are elastic and $M \gg m$



2D Elastic: Nuclear scattering.

A particle of unknown mass M is initially at rest. A particle of known mass m is "shot" against it with initial momentum p_i. After the collision, the momentum of the particle of known mass is measured again, and it is p_f.

If the collision is elastic, that's all we need to determine M and the final momentum of the target, P.



3 unknowns: M, P_x, P_y

3 equations: conservation of momentum in the x direction

conservation of momentum in the y direction

conservation of kinetic energy

We can write the kinetic energy in terms of momentum:

$$KE = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = \frac{p^2}{2m}$$

$$\vec{p}_i = \vec{p}_f + \vec{P} \quad \text{(2 equations)} \longrightarrow \vec{P}^2 = P^2 = (\vec{p}_i - \vec{p}_f)^2$$

$$\frac{p_i^2}{2m} = \frac{p_f^2}{2m} + \frac{P^2}{2M} \longrightarrow P^2 = 2M\left(\frac{p_i^2}{2m} - \frac{p_f^2}{2m}\right) =$$

$$= \frac{M}{m}(p_i^2 - p_f^2)$$

$$\boxed{\vec{P} = \vec{p}_i - \vec{p}_f}$$

$$\boxed{M = m\frac{(\vec{p}_i - \vec{p}_f)^2}{(p_i^2 - p_f^2)}}$$

By bombarding an unknown particle with a known projectile and measuring the initial and final momentum, we can find the mass of the target!

Used in atomic, nuclear and elementary particle physics.