Lecture 18

Linear Momentum and Impulse

Momentum and Newton's Second Law

$$\vec{F}_{net} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}$$

Linear momentum $\vec{p} \equiv m\vec{v}$

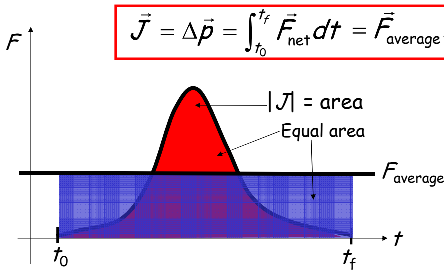
Newton's second law in terms of linear momentum: $\vec{F}_{net} = \frac{d\vec{p}}{dt}$

A net force is a transfer of linear momentum. $\Delta\vec{p} = \int \vec{F}_{net}\,dt$

Impulse

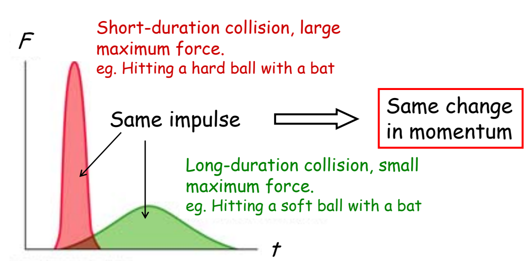
The change in linear momentum for a certain time interval is called the <u>impulse $\vec{J}$</u> (something like "how much of an effect did the force have by being applied during a given time interval ").

$$\vec{J} = \Delta\vec{p} = \int_{t_0}^{t_f} \vec{F}_{net}\,dt = \vec{F}_{average}\Delta t$$



$$\vec{F}_{average} = \frac{\vec{J}}{\Delta t} = \frac{\Delta\vec{p}}{\Delta t}$$

$$\left(\vec{F} = \frac{d\vec{p}}{dt}\right)$$



Units of Momentum and Impulse

SI units

$$\vec{p} \equiv m\vec{v} \longrightarrow \textbf{kg m/s}$$

It's the same!

$$\vec{J} = \Delta\vec{p} = \int \vec{F}_{net}\,dt \longrightarrow \textbf{N s}$$

ACT: Egg tossing

In an egg-tossing contest, two people toss a raw egg back and forth. After each successful toss, each person takes a step back. Catching the egg without breaking it becomes harder and harder. Usually the trick is moving your hand down with the egg when you receive it. This works better because:

A. It decreases the change in momentum.

B. It decreases the impulse.

C. It decreases the force on the egg.

- If the flying egg has speed v, the change in momentum is:

  $\Delta p = 0 - mv = -mv$ (independent of how you catch it)

- The impulse is just the same!

  $J = \Delta p = -mv$

- But by moving your hand with the egg, you are increasing the time interval over which this Δp must take place. So the average force on the egg

  $$F_{ave} = \frac{\Delta p}{\Delta t} \quad \text{decreases.}$$

By the way: Catching the egg is harder and harder because its speed becomes larger (and the required change in momentum, too), so exerting a small force becomes harder as well.

Example:

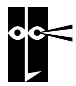
Impulse

Jiggs is completely missing the point…

If the brick has a mass of 1 kg and has been dropped from 10 m, what is the average force on Jerry's head during the 10 ms it is in contact with his head?

Right before hitting Jerry's head, the speed of the brick is:

$$mgh = \frac{1}{2}mv^2$$
$$v = \sqrt{2gh} = \sqrt{2(9.8\text{ m/s}^2)(10\text{ m})} = 14\text{ m/s}$$



DEMO: Karate boards

If the brick is completely stopped by his head, the change in momentum is:

$$\Delta p = 0 - mv = -(1\text{ kg})(14\text{ m/s}) = -14\text{ kg m/s}$$

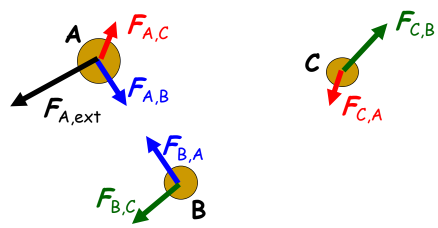
Therefore, the average force is:

$$\left|F_{\text{average}}\right| = \frac{|J|}{\Delta t} = \frac{|\Delta p|}{\Delta t} = \frac{14\text{ kg m/s}}{0.01\text{ s}} = 1400\text{ N}$$

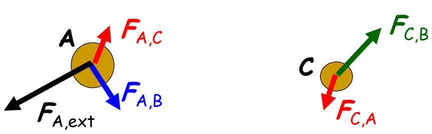
Equivalent to 315 lb. Ouch!!

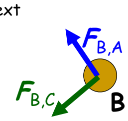
EXAMPLE: System of three particles

Consider a system of 3 particles that are exerting forces (of whatever nature) on one another, and there's also an external force on A (done by some external agent):



A: $$\frac{d\vec{p}_A}{dt} = \vec{F}_{A,B} + \vec{F}_{A,C} + \vec{F}_{A,ext}$$





C: $$\frac{d\vec{p}_C}{dt} = \vec{F}_{C,A} + \vec{F}_{C,B}$$

B: $$\frac{d\vec{p}_B}{dt} = \vec{F}_{B,A} + \vec{F}_{B,C}$$

$$\frac{d\vec{p}_A}{dt} = \vec{F}_{A,B} + \vec{F}_{A,C} + \vec{F}_{A,ext}$$
$$\frac{d\vec{p}_B}{dt} = \vec{F}_{B,A} + \vec{F}_{B,C}$$
$$\frac{d\vec{p}_C}{dt} = \vec{F}_{C,A} + \vec{F}_{C,B}$$

$$\frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt} + \frac{d\vec{p}_C}{dt} = $$
$$= \cancel{\vec{F}_{A,B}} + \cancel{\vec{F}_{A,C}} + \vec{F}_{A,ext} + \cancel{\vec{F}_{B,A}} + \cancel{\vec{F}_{B,C}} + \cancel{\vec{F}_{C,A}} + \cancel{\vec{F}_{C,B}}$$
$$= \vec{F}_{A,ext}$$

Define <u>total linear momentum</u> for the system:

$$\vec{p}_{\text{total}} = \sum_i \vec{p}_i = \sum_i m\vec{v}_i$$

$$\frac{d\vec{p}_{total}}{dt} = \vec{F}_{ext}$$

Conservation of momentum

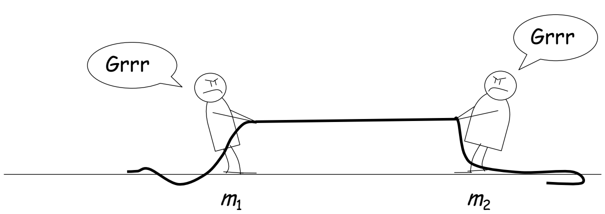
The really important case:

If $\vec{F}_{ext} = 0$, $\quad \dfrac{d\vec{p}_{total}}{dt} = 0$

$$\vec{p}_{\text{initial}} = \vec{p}_{\text{final}}$$

EXAMPLE: Tug of war… on ice

Two guys of masses m_1 = 75 kg and m_2 = 90 kg pull on both ends of a rope on an ice rink. After a couple of seconds, the thin one is moving at 0.2 m/s. What is the speed of the big one?



Two guys of masses m_1 =75 kg and m_2=90 kg pull on both ends of a rope on an ice rink. After a couple of seconds, the thin one is moving at 0.2 m/s. What is the speed of the big one?

No friction

No net vertical force

⟶ F_{ext} = 0 ⟶ p_{total} **is conserved**

$$p_{1,i} + p_{2,i} = p_{1,f} + p_{2,f} \longrightarrow 0 + 0 = m_1 v_{1,f} + m_2 v_{2,f}$$

$$v_{2,f} = -\frac{m_1}{m_2} v_{1,f} = -\frac{75\text{ kg}}{90\text{ kg}} 0.2\text{ m/s} = -0.17\text{ m/s}$$

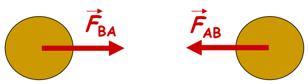
Changes in linear momentum:

$$\Delta p_1 = m_1\left(v_{1,f} - v_{1,i}\right) = (75\text{ kg})(0.2\text{ m/s - }0) = 15\text{ kg m/s}$$

$$\Delta p_2 = m_2\left(v_{2,f} - v_{2,i}\right) = (90\text{ kg})(-0.17\text{ m/s - }0) = -15\text{ kg m/s}$$

The force produced a <u>momentum tranfer</u>.

Conservation of momentum in a 2-particle system

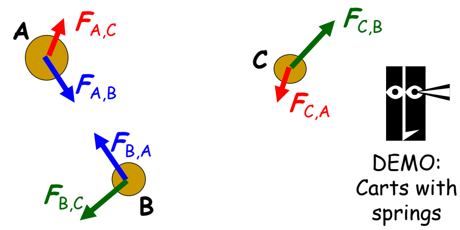


$$\vec{F}_{BA} = -\vec{F}_{AB}$$

$$\therefore \quad \frac{d\vec{p}_A}{dt} = -\frac{d\vec{p}_B}{dt}$$

The interaction means there is an exchange of linear momentum between two objects.

This is general for any isolated system (a system which is not subject to a net external force).

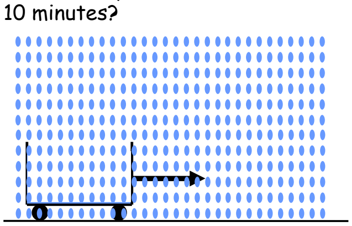


DEMO: Carts with springs

Each interactions within the system represents momentum flows between the particles but the total momentum of the system remains constant.

Example: Rain

Rain falls vertically into a 10-kg open cart rolling at 2.0 m/s along a straight horizontal track with negligible friction. The cart fills at a rate of 0.1 liters per minute. What is the speed of the cart after 10 minutes?



External horizontal force = 0 $\longrightarrow$ $P_x = mv$ is constant

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Rain falls vertically into a 10-kg open cart rolling at 2.0 m/s along a straight horizontal track with negligible friction. The cart fills at a rate of 0.1 liters per minute. What is the speed of the cart after 10 minutes?

How much water is inside the cart after 10 minutes?

$$10 \text{ minutes} \frac{0.1 \text{ liters}}{1 \text{ minute}} \frac{1 \text{ kg water}}{1 \text{ liter water}} = 1 \text{ kg water}$$

Linear momentum is conserved.

$$m_i v_i = m_f v_f$$

$$(10 \text{ kg})(2.0 \text{ m/s}) = (10 \text{ kg} + 1 \text{ kg}) v_f$$

$$v_f = 1.8 \text{ m/s}$$