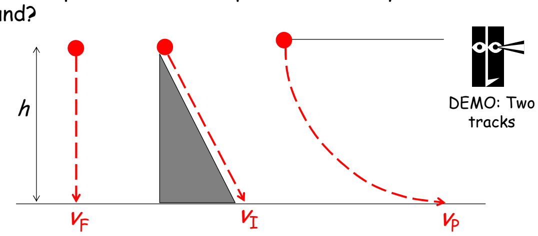
Lecture 16

Conservative and Non-Conservative Forces

Examples.

ACT: Falling objects

Three objects of mass m are dropped from a height h. One falls straight down, one slides down a frictionless incline and one swings at the end of a pendulum. What is the relationship between their speeds when they reach the ground?



A. $v_F > v_I > v_P$ B. $v_F > v_P > v_I$ C. $v_F = v_I = v_P$

In all three cases, the only force doing work is gravity → mechanical energy is conserved.

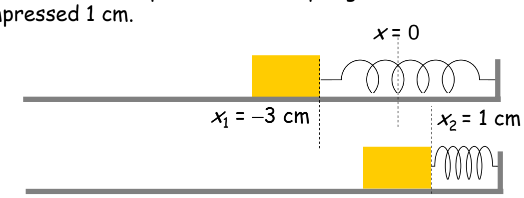
$$E_i = 0 + mgh$$

$$E_f = \frac{1}{2}mv^2 + 0$$

→ Same final speed

Oscillations

A glider of mass m = 0.5 kg on a horizontal frictionless surface is attached to a spring with k = 200 N/m. The glider is pulled 3 cm away from the equilibrium position and released. Find its speed when the spring has been compressed 1 cm.



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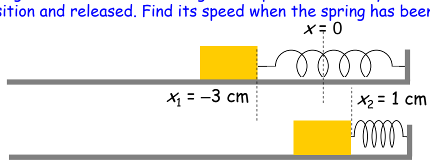


DEMO: Glider on a track

$$E_1 = \frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 \quad (\tfrac{1}{2}mv_1^2 = 0) \qquad E_2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2$$

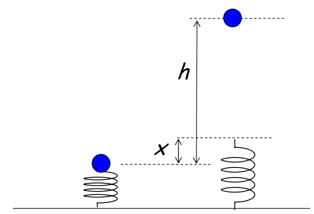
$$\frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2$$

Careful with the units

$$v_2 = \sqrt{\frac{k}{m}\left(x_1^2 - x_2^2\right)} = \sqrt{\frac{(200\ \text{N/m})}{0.5\ \text{kg}}\left(0.03^2 - 0.01^2\right)\text{m}^2} = 0.57\ \text{m/s}$$

EXAMPLE: Vertical spring

A 50-g ball is shot by a vertical spring compressed over a distance x = 2.0 cm. It reaches a height h = 2.5 m above the initial position. Determine the spring constant k.



Mechanical energy of the ball:

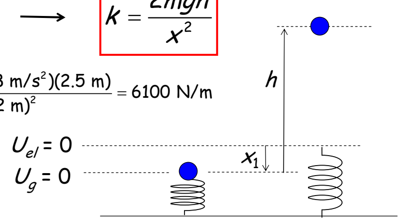
$$E = \frac{1}{2}mv^2 + mgh + \frac{1}{2}kx^2$$ (with the appropriate choice of zero potential energies, see figure)

Before the shot: $E = \frac{1}{2}kx^2 \qquad (v_{initial} = 0)$

At the top: $E = mgh \qquad (v_{top} = 0)$

$$\frac{1}{2}kx^2 = mgh \longrightarrow \boxed{k = \frac{2mgh}{x^2}}$$

$$k = \frac{2(0.05\ \text{kg})(9.8\ \text{m/s}^2)(2.5\ \text{m})}{(0.02\ \text{m})^2} = 6100\ \text{N/m}$$



DEMO: Hopper-popper and ball

Mechanical energy with non-conservative forces.

In a system where only conservative forces are doing work, mechanical energy is conserved: $\Delta E = 0$

If both conservative and non-conservative forces are doing work:

$$W_{net} = \Delta K \longrightarrow -\Delta U + W_{n\text{-}c} = \Delta K \longrightarrow \Delta(K + U) = W_{n\text{-}c}$$

$$W_{net} = -\Delta U + W_{n\text{-}c}$$

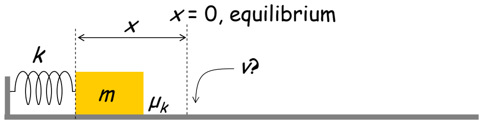
Mechanical energy is not conserved

$$\boxed{\Delta E = W_{n\text{-}c}}$$

Example: Spring with friction

A 5.00 g block is pushed against a spring with k = 8.00 N/m. The spring is initially compressed 5.00 cm and then released. The coefficient of kinetic friction between the block and the table is 0.600. What is the speed of the block at x = 0?

A. 1.75 m/s B. 1.85 m/s C. 1.95 m/s D. 2.00 m/s E. 2.05 m/s



Because friction (a non-conservative force) is doing work, mechanical energy is NOT conserved: $\Delta E = W_{\text{friction}}$

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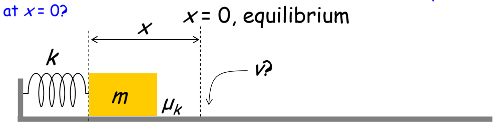


$$E_{\text{initial}} = 0 + \frac{1}{2}kx^2 \qquad E_{\text{final}} = \frac{1}{2}mv^2 + 0 \qquad W_{\text{friction}} = -\mu_k mgx$$

$$\frac{1}{2}mv^2 - \frac{1}{2}kx^2 = -\mu_k mgx \longrightarrow \boxed{v = \sqrt{\frac{kx^2}{m} - 2\mu_k gx}}$$

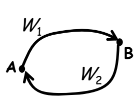
$$v = \sqrt{\frac{(8.00 \text{ N/m})(0.05 \text{ m})^2}{0.005 \text{ kg}} - 2(0.600)(9.81 \text{ m/s}^2)(0.05 \text{ m})} = 1.85 \text{ m/s}$$

Answer B

Conservative forces.

The work done by a conservative force does not depend on the trajectory.

- A potential energy function can be defined.
- The work by a conservative force in a closed path is zero.



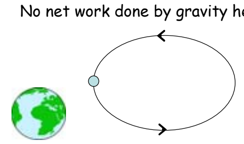
$$W_{\text{total}} = W_1 + W_2 = W_1 - W_1 = 0$$

- Mechanical energy is conserved. A conservative force allows for the reversible storage of energy.

Examples:
- All fundamental forces of nature (in 221: gravity and electric force)
- Elastic forces.

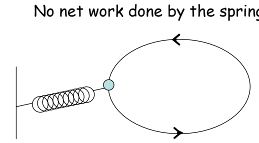
Gravity: Conservative

No net work done by gravity here



Elastic Forces: Conservative

No net work done by the spring here



Non-conservative forces

Non-conservative force = force that is not conservative.

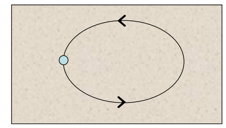
- The work done by a non-conservative force depends on the trajectory.
- The work done along a closed trajectory is, in general, not zero.
- A potential energy function cannot be defined.
- Mechanical energy is "lost" or "dissipated" ($W_{n\text{-}c} < 0$) or gained through a non-reversible process ($W_{n\text{-}c} > 0$)

Examples:

- Kinetic Friction
- Fluid drag

  $W_{n\text{-}c} < 0$, mechanical energy is "dissipated" and becomes internal energy of surrounding objects.

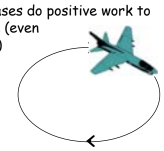
Friction does negative work when an object is dragged in a closed path over sand paper.



- *Gas expansion*

  $W_{n\text{-}c} > 0$, internal energy of the gas is transformed into mechanical energy.

The expanding gases do positive work to impulse the plane (even in a closed circle)

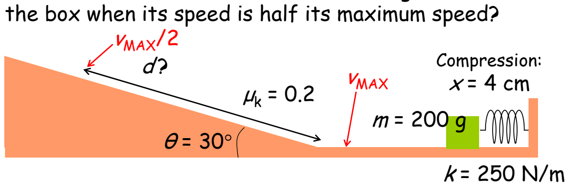


- Any other process that converts between mechanical energy and internal energy (chemical energy; nuclear energy…)

DEMO: (Ir)reversible processes

EXAMPLE: Spring and incline

In the system below, a 200-g box is pushed 4 cm against a spring with k = 250 N/m and released. The box slides along a frictionless horizontal surface and then up an incline which makes an angle of 30° with respect to the horizontal. The coefficient of kinetic friction between the box and the incline is 0.2. How far along the incline is the box when its speed is half its maximum speed?



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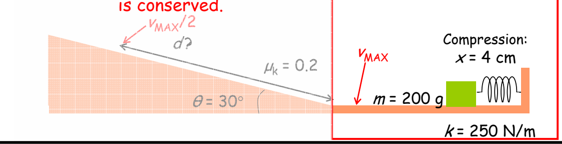
What is v_{MAX}?

$$E_{\text{initial}} = 0 + \frac{1}{2}kx^2$$
$$E_{\text{final}} = \frac{1}{2}mv^2 + 0$$
$$E_{\text{initial}} = E_{\text{final}} \rightarrow v_{(MAX)} = x\sqrt{\frac{k}{m}} = (0.04\text{ cm})\sqrt{\frac{250\text{ N/m}}{0.2\text{ kg}}} = 1.4\text{ m/s}$$

In this part of the motion, mechanical energy $E = K + U_{\text{elastic}}$ is conserved.



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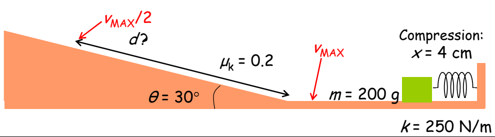
For the whole process, mechanical energy $E = K + U_g + U_{\text{elastic}}$ is **not** conserved due to friction: $\Delta E = W_{\text{friction}}$

$$\left.\begin{aligned} E_{\text{initial}} &= 0 + 0 + \frac{1}{2}kx^2 \\ E_{\text{final}} &= \frac{1}{2}m\left(\frac{v_{\text{MAX}}}{2}\right)^2 + mgh + 0 \\ W_{\text{friction}} &= -f_k d \end{aligned}\right\} \frac{1}{2}m\left(\frac{v_{\text{MAX}}}{2}\right)^2 + mgh - \frac{1}{2}kx^2 = -f_k d$$



$$\frac{1}{2}m\left(\frac{v_{\text{MAX}}}{2}\right)^2 + mgh - \frac{1}{2}kx^2 = -f_k d \quad \text{with} \begin{cases} h = d\sin\theta \\ f_k = \mu_k mg\cos\theta \\ v_{(\text{MAX})} = x\sqrt{\dfrac{k}{m}} \end{cases}$$

$$\frac{1}{8}kx^2 + mgd\sin\theta - \frac{1}{2}kx^2 = -\mu_k mgd\cos\theta$$

$$d = \frac{3kx^2}{8mg(\sin\theta + \mu_k\cos\theta)} = \frac{3(250\text{ N/m})(0.04\text{ cm})^2}{8(0.2\text{ kg})(9.8\text{ m/s}^2)(\sin 30° + 0.2\cos 30°)}$$
$$= 0.114\text{ m} = 11.4\text{ cm}$$