Lecture 14

Work: Varying forces and curved trajectories

Power

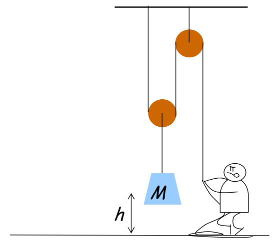
ACT: Zero net work

The system of pulleys shown below is used to lift a bag of mass M at constant speed a distance h from the floor. What is the work done by the person?

A. Mgh

B. $\frac{1}{2}Mgh$

C. $2Mgh$



$W_{net} = \Delta KE = 0$ (constant speed)

$W_T = -W_g = -(-Mgh)$

The force by the person is $Mg/2$ (see lecture 10), but he needs to pull on a length of rope of $2h$.

The story so far:

Kinetic energy $K = \frac{1}{2}mv^2$

Work by a constant force, along a straight path:

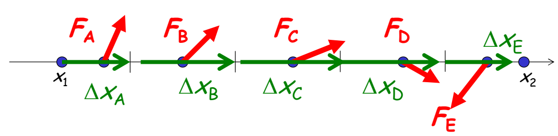
$$W = \vec{F} \cdot \Delta\vec{r} = F\Delta r\cos\theta = F_{//}\,\Delta r$$

Work/Kinetic energy theorem

$$W_{net} = \Delta K$$

Work by non-constant force, with straight line trajectory

An object moves along the x-axis from point x_1 to x_2. A non-constant force is applied on the object. What is the work done by this force?

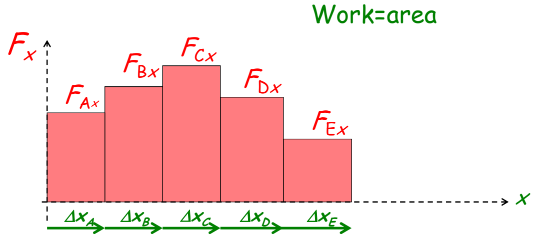


The journey is divided up into a series of segments over which the force is constant.

The total work is the sum of the works for each of the intervals:

$$W = F_{Ax}\Delta x_A + F_{Bx}\Delta x_B + F_{Cx}\Delta x_C + F_{Dx}\Delta x_D + F_{Ex}\Delta x_E$$

Work=area



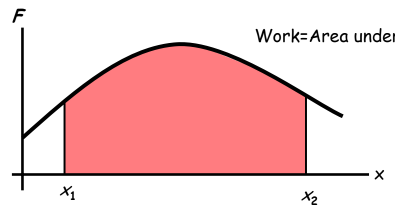
$$W = F_{Ax}\Delta x_A + F_{Bx}\Delta x_B + F_{Cx}\Delta x_C + F_{Dx}\Delta x_D + F_{Ex}\Delta x_E$$

This is still an approximation, because the force is not really constant in each interval…

The calculation is really good in the limit where the intervals are very small, the sum becomes an integral:

$$W = \int_{x_1}^{x_2} F_x(x)dx$$

Work by non-constant force, with straight line trajectory

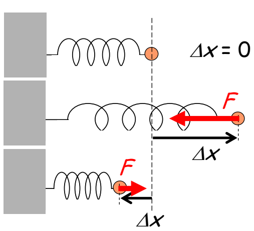


Springs

Hooke's law: The force exerted by a spring is proportional to the distance the spring is compressed/stretched from the relaxed position.

$$F_x = -k\,\Delta x$$

k = spring constant

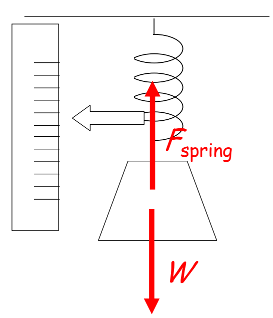


$\Delta x = 0$ $F = 0$

$\Delta x > 0$ $F < 0$

$\Delta x < 0$ $F > 0$

Hooke's law is the basis of traditional scales.



$$F - W = 0$$

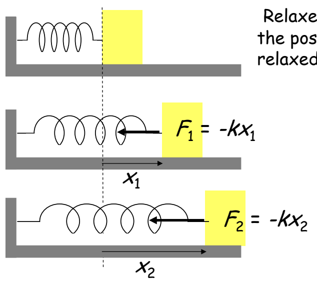
$$k\Delta x - mg = 0$$

$$m = \frac{k}{g}\Delta x$$

DEMO: Springs and weights

Stretching a Spring

What is the work done by the spring on the block as the tip is pulled from x_1 to x_2?



Relaxed position (Define $x = 0$ as the position of the end of the relaxed spring. Then $F = -kx$)

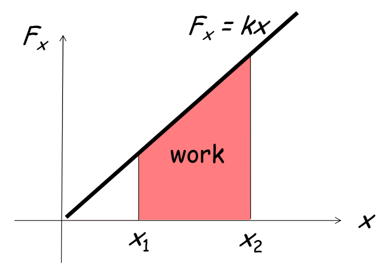
Varying force: an integral is required.

What is the work done by a spring as the tip is pulled from x_1 to x_2?

$$W_{\text{by spring}} = \int_{x_1}^{x_2} \vec{F}(x)\cdot d\vec{r} = -\int_{x_1}^{x_2} kx\,dx = -\frac{1}{2}kx^2\Big]_{x_1}^{x_2} = -\left(\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2\right)$$

What is the work done **on** the spring as the tip is pulled from x_1 to x_2?

$$W_{\text{external}} = -W_{\text{by spring}} = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$



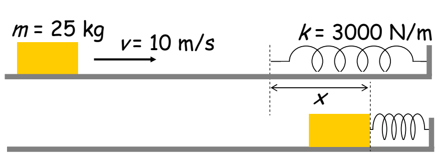
If $x_2 > x_1$ (stretch), $W_{\text{external}} > 0$

We are adding energy to the spring

A stretched or compressed spring stores energy (elastic energy).

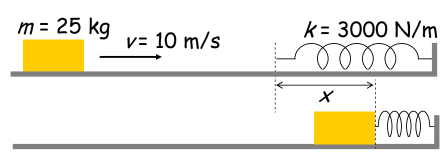
Example: Box and spring

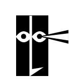
A box of mass $m = 25$ kg slides on a horizontal frictionless surface with an initial speed $v_0 = 10$ m/s. How far will it compress the spring before coming to rest if $k = 3000$ N/m?



A. 0.50 m B. 0.63 m C. 0.75 m

D. 0.82 m E. 0.91 m

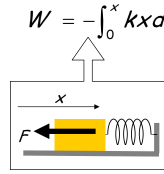




DEMO: Horizontal spring

Use the work-kinetic energy theorem: $W = \Delta KE$

$$W = -\int_0^x kx\,dx = -\frac{1}{2}kx^2 \qquad \Delta KE = 0 - \frac{1}{2}mv^2$$



$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

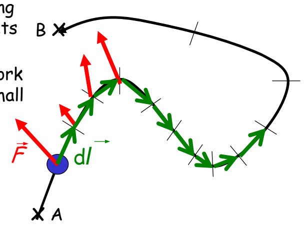
$$x = \sqrt{\frac{mv^2}{k}} = \sqrt{\frac{(25\text{ kg})(10\text{ m/s})^2}{3000\text{ N/m}}} = 0.91\text{ m}$$

Answer E

Work on curved trajectories

A particle moves from A to B along this path while a varying force acts upon it.

Again, we can approximate the work by considering breaking it into small displacements $d\vec{l}$.



If the intervals are very small, the trajectory is straight and the force is constant, so the work done by a force is:

$$dW = \vec{F}\cdot d\vec{l}$$

We need to add -integrate- all the contributions:

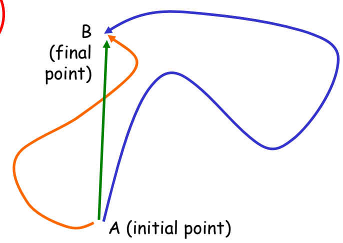
$$W_{A\to B} = \int_A^B \vec{F}\cdot d\vec{l}$$

The line integral

$$W_{A\to B} = \int_A^B \vec{F}\cdot d\vec{l}$$

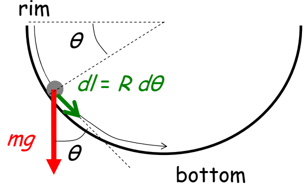
path

For three different paths to move from A to B, a force will in general do different works!



Example: Spherical bowl

Find the work done by gravity on a pebble of mass m as it rolls from the rim to the bottom of a spherical bowl of radius R.



$$W = \int_{top}^{bottom} m\vec{g}\cdot d\vec{l}$$

$$= \int_{\theta=0}^{\theta=\pi/2} mg\ Rd\theta\ \cos\theta$$

$$= mgR\int_0^{\pi/2}\cos\theta\ d\theta$$

$$= mgR\sin\theta\Big|_0^{\pi/2}$$

$$= mgR$$

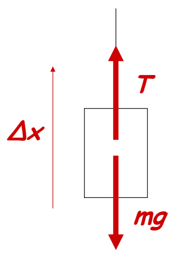
ACT: Two elevators

Two elevators A and B carry each a load of mass m from the first floor to the third floor of a building at constant speeds, but A is twice as fast as B. The work done by the cable tension (ie, the energy produced by the engine) is:

A. Larger for A

B. Larger for B

C. The same for both



$W_{net} = \Delta KE$

$W_T + W_g = 0$

$W_T = -W_g = -(-mg\Delta x)$

$\boxed{W_T = mg\Delta x} > 0$

for both elevators

The engine is <u>adding</u> energy.

Ok, so what's the difference between the two elevators?

Power

The machinery in elevator A has more power: it is doing the same work in less time (or more work per unit time).

Average power:

$$P_{average} = \frac{W}{\Delta t} \qquad P_{average} = \frac{\vec{F}\cdot\Delta\vec{r}}{\Delta t} = \vec{F}\cdot\vec{v}_{average}$$

Instantaneous power:

$$P = \frac{dW}{dt} \qquad P = \frac{\vec{F}\cdot d\vec{r}}{dt} = \vec{F}\cdot\vec{v}$$

Units of power

SI:	Watt	1 W = 1 J/s
Other:	Horsepower	1 hp = 746 W

Kilowatt-hour (kWh) is a unit of energy or work:

$$1\text{ kWh} = 1\text{ kW}\cdot\text{h}\frac{1000\text{ W}}{1\text{ kW}}\frac{3600\text{ s}}{1\text{ h}} = 3.6\times10^{6}\ \text{~~Ws~~ J}$$