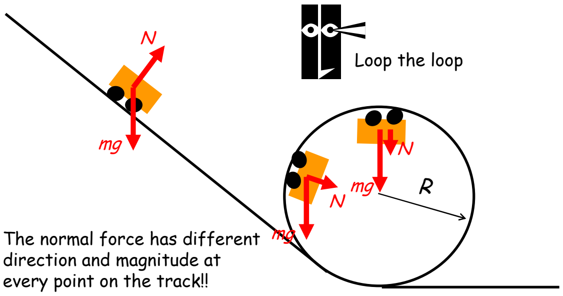
Lecture 13

Work and the Work/Kinetic Energy Theorem

Motivation to go beyond Newton's laws

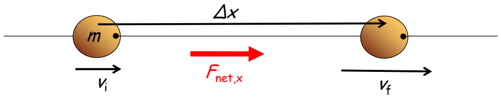


The normal force has different direction and magnitude at every point on the track!!

Writing and solving Newton's laws can be a nasty experience… (see appendix of lecture 15)

Playing around with Newton's 2nd law.

Consider the motion of a bead of mass *m* on a straight wire pushed by a constant net force F_x parallel to the wire, along a displacement Δx:



Force produces acceleration: $F_x = ma_x$

Acceleration produces change in speed: $2a_x \Delta x = v_f^2 - v_i^2$

$$2\frac{F_x}{m}\Delta x = v_f^2 - v_i^2$$

WORK (*W*) done by force *F* over displacement Δx

$$\boxed{F_{\text{net},x}\Delta x} = \boxed{\frac{1}{2}mv_f^2} - \boxed{\frac{1}{2}mv_i^2}$$

Change in KINETIC ENERGY (*K*) of the bead

This is an expression of the "effectiveness" of the force.

$$F_{\text{net},x}\Delta x = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$\boxed{W_{\text{net}} = \Delta K}$$

Work/Kinetic Energy Theorem

Of how a force applied over a distance…

…changes something in the system

Work

The "external agent" that changes the amount of kinetic energy (the state) in the system

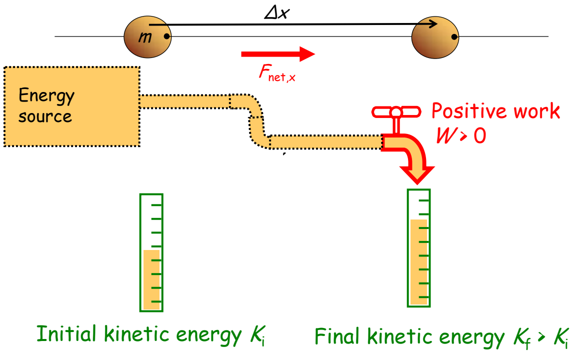
$$W = F_x \Delta x$$

Kinetic energy

The "internal" quantity (state) of the system

$$K = \frac{1}{2}mv^2$$

Work is energy being transferred



Units for work and energy:

SI:

Joule	1 J = 1 N · m

Other common units:

Kilowatt-hour	KWh
calorie	1 cal = 4.184 J
Not to be confused with: Calorie (or food calorie)	1 Cal = 1000 cal

Energy

Many types of energy:

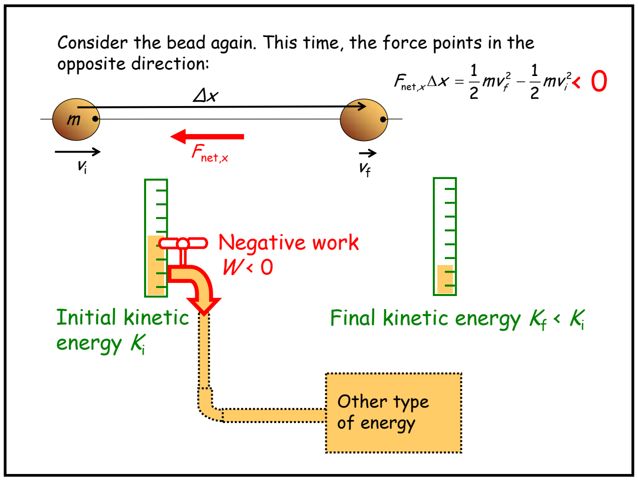
- kinetic energy
- electric energy
- internal –thermal- energy
- elastic energy
- chemical energy
- Etc.

Energy is transferred and **transformed** from one type to another and is never destroyed or created.

What is work?

The definition of work $W = F\Delta x$ corresponds to the intuitive idea of effort:

- More massive object will require more work to get to the same speed (from rest)
- For a given mass, getting to a higher speed (from rest) requires more work
- If we push for a longer distance, it's more work.
- It takes the same work to accelerate the object to the right as to the left (both displacement and force reverse)



Consider the bead again. This time, the force points in the opposite direction:

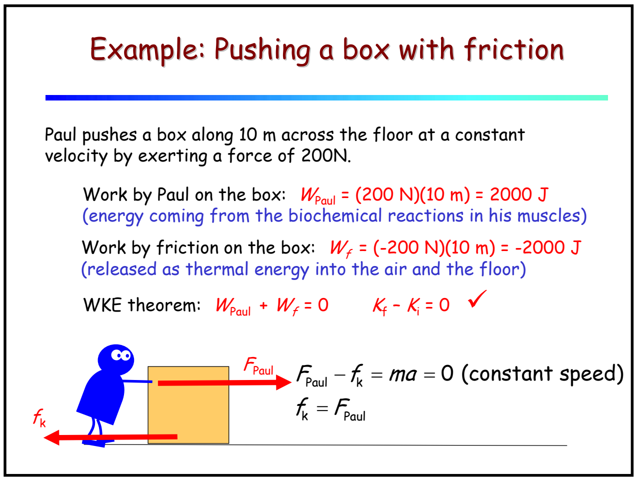
$$F_{\text{net},x}\Delta x = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 < 0$$

Negative work $W < 0$

Initial kinetic energy K_i

Final kinetic energy $K_f < K_i$

Other type of energy



Example: Pushing a box with friction

Paul pushes a box along 10 m across the floor at a constant velocity by exerting a force of 200N.

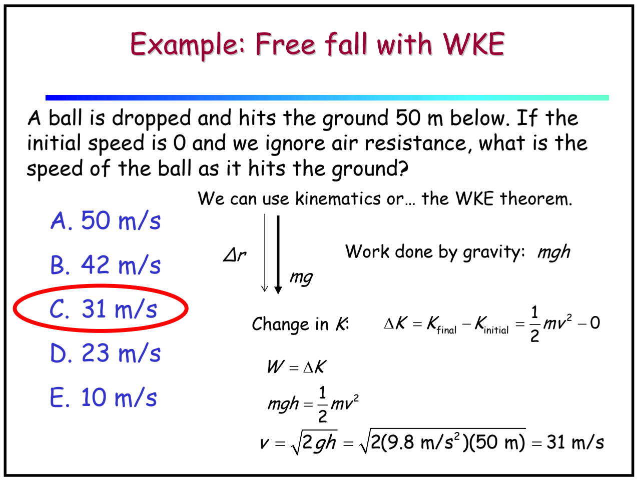
Work by Paul on the box: W_{Paul} = (200 N)(10 m) = 2000 J
(energy coming from the biochemical reactions in his muscles)

Work by friction on the box: W_f = (-200 N)(10 m) = -2000 J
(released as thermal energy into the air and the floor)

WKE theorem: $W_{\text{Paul}} + W_f = 0$ $K_f - K_i = 0$ ✓

$F_{\text{Paul}} - f_k = ma = 0$ (constant speed)

$f_k = F_{\text{Paul}}$



Example: Free fall with WKE

A ball is dropped and hits the ground 50 m below. If the initial speed is 0 and we ignore air resistance, what is the speed of the ball as it hits the ground?

- A. 50 m/s
- B. 42 m/s
- [✓ C. 31 m/s]
- D. 23 m/s
- E. 10 m/s

We can use kinematics or… the WKE theorem.

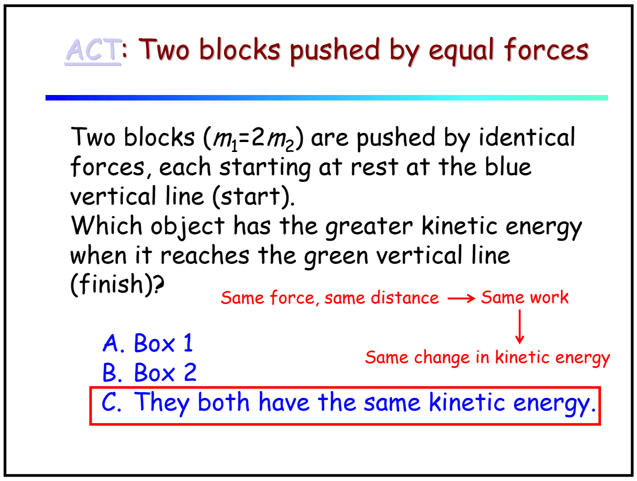
Work done by gravity: mgh

Change in K: $\Delta K = K_{\text{final}} - K_{\text{initial}} = \frac{1}{2}mv^2 - 0$

$$W = \Delta K$$

$$mgh = \frac{1}{2}mv^2$$

$$v = \sqrt{2gh} = \sqrt{2(9.8\text{ m/s}^2)(50\text{ m})} = 31\text{ m/s}$$



ACT: Two blocks pushed by equal forces

Two blocks (m_1=2m_2) are pushed by identical forces, each starting at rest at the blue vertical line (start).
Which object has the greater kinetic energy when it reaches the green vertical line (finish)?

Same force, same distance ⟶ Same work
↓
Same change in kinetic energy

- A. Box 1
- B. Box 2
- [✓ C. They both have the same kinetic energy.]

What if the force does not point in the direction of motion?

Recall the v² equation in multi-dimensions: $2\vec{a}\cdot\Delta\vec{r} = v_f^2 - v_i^2$

$$\vec{a}\cdot\Delta\vec{r} = \frac{1}{m}\boxed{\vec{F}\cdot\Delta\vec{r}}$$

Work by a constant force along a straight path:

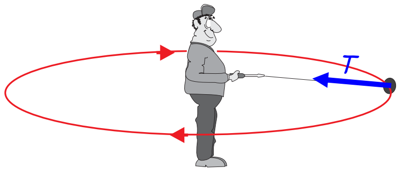
$$\boxed{W = \vec{F}\cdot\Delta\vec{r} = F\Delta r\cos\theta = F_{//}\,\Delta r}$$

For work, only the part of the force that is in the direction of displacement "matters" (= can change speed)

(= can change kinetic energy)

Examples of Non-Work

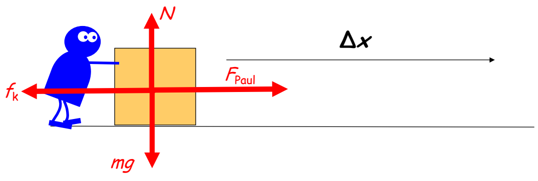
This person is not doing any work on the rock since the centripetal force is perpendicular to the direction that the stone is moving.



Examples of Non-Work

The weight of the box that Paul pushes along a horizontal surface does no work since the weight is perpendicular to the motion

The normal by the floor does no work either.



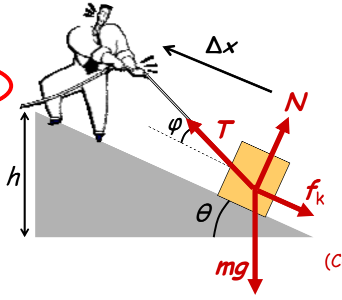
ACT: Ramp

A person pulls a 10-kg block up a ramp of height h = 10 m and angle θ = 10° at constant speed. The rope makes an angle φ = 15° with the ramp and has a tension of 50 N. How many forces are doing work on the box?

A. 2

[✓ B. 3]

C. 4



$W_T > 0$

$W_f < 0$

$W_g < 0$

$W_N = 0$

$W_{Net} = 0$

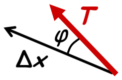
(Constant speed)

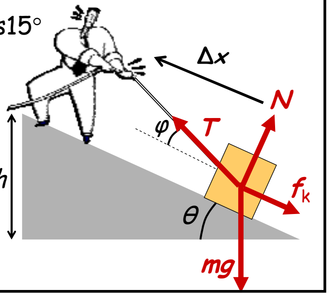
A person pulls a 10-kg block up a ramp of height h = 10 m and angle θ = 10° at constant speed. The rope makes an angle φ = 15° with the ramp and has a tension of 50 N. Find the work done by all the forces.

1. Work by the tension:

$$W_T = T\Delta x\cos\varphi$$
$$= (50\ \text{N})(57.6\ \text{m})\cos 15^\circ$$
$$= 2780\ \text{J}$$

$$\Delta x = \frac{h}{\sin\theta} = \frac{10\ \text{m}}{\sin 10^\circ} = 57.6\ \text{m}$$





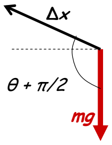
A person pulls a 10-kg block up a ramp of height h = 10 m and angle θ = 10° at constant speed. The rope makes an angle φ = 15° with the ramp and has a tension of 50 N. Find the work done by all the forces.

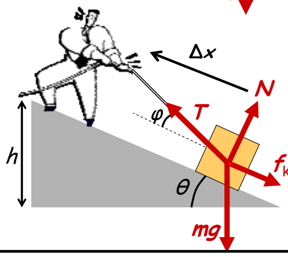
2. Work by gravity:

$$W_g = mg\Delta x\cos\left(\theta + \frac{\pi}{2}\right)$$
$$= -mg\Delta x\sin\theta$$
$$= -mgh$$
$$= -980\ \text{J}$$

Of course!

$-h$ is the component of the displacement in the direction of weight!





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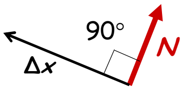
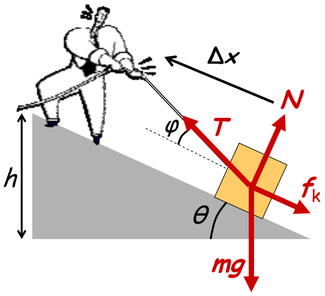
3. Work by the normal:

W_N =0





A person pulls a 10-kg block up a ramp of height h = 10 m and angle θ = 10° at constant speed. The rope makes an angle φ = 15° with the ramp and has a tension of 50 N. Find the work done by all the forces.

4. Work by the friction:

Two options:

1. Find f_k with Newton's laws and use it to find the work (long)
2. Use the WKE theorem (smart): $W_{net} = \Delta KE = 0$

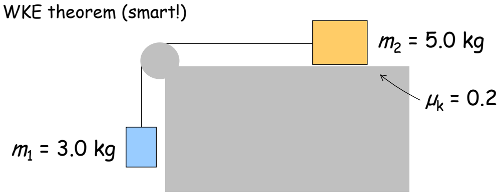
$$W_{net} = W_g + W_N + W_T + W_f = 0$$
$$W_f = -\left(W_g + W_N + W_T\right)$$
$$= -(-980\ \text{J}+0+2780\ \text{J}) = -1800\ \text{J}$$

Example: Systems with several objects

What is the speed of the system after box 1 has fallen for 30 cm?

Two approaches:

1. Use Newton's laws to find acceleration; use kinematics to find speed. (long!!)
2. Use WKE theorem (smart!)



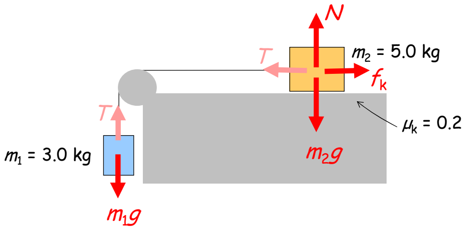
What is the speed of the system after box 1 has fallen for $\underset{= x}{\underline{\text{30 cm}}}$?

External forces doing work: $m_1 g$, f_k

(The tensions are internal forces: their net work is zero)

$$\begin{aligned} W_{\text{net}} &= m_1 g x - f_k x \\ &= m_1 g x - \mu_k m_2 g x \end{aligned}$$

$$\Delta KE = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 - 0$$



What is the speed of the system after box 1 has fallen for 30 cm?

$$W_{\text{net}} = m_1 g x - \mu_k m_2 g x \qquad \Delta KE = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 - 0$$

$$\left(m_1 - \mu_k m_2\right) g x = \frac{1}{2}\left(m_1 + m_2\right) v^2$$

$$v = \sqrt{\frac{m_1 - \mu_k m_2}{m_1 + m_2} 2 g x} = \sqrt{\frac{3 - 0.2 \times 5}{3 + 5} 2(9.8)(0.3)} = 1.2 \text{ m/s}$$

m_1 = 3.0 kg

m_2 = 5.0 kg

μ_k = 0.2

x = 0.3 m