# Lecture 12

## Applying Newton's Laws

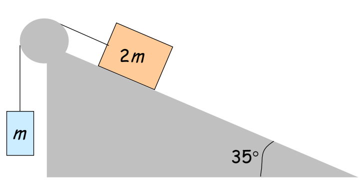
## Goals of Today's Lecture

Being able to apply Newton's laws to any problem that is not insane.

## EXAMPLE: Incline and pulley, with friction

Same system, but $\mu_S$ = 0.2 and $\mu_k$ = 0.1

What happens when the system is released?




Does the system move at all?

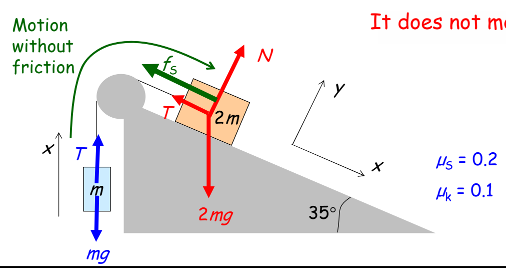
Net force, along the direction of motion, without friction:

$$F_{net} = 2mg\sin 35° - mg = 0.15mg$$

Maximum static friction force:

$$f_{S,MAX} = \mu_S N = \mu_S 2mg\cos 35° = 0.33mg$$

It does not move!



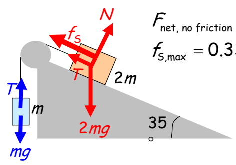
## ACT: Magnitude of friction

What is the magnitude of the static force in the system we just studied?

A. 0.15 *mg*

B. 0.33 *mg*

C. 0.40 *mg*




$F_{\text{net, no friction}} = 0.15mg$

$f_{S,\max} = 0.33mg$

For the system to be at rest, $f_S$ should cancel out the resultant of the remaining forces (the net force without friction).

Option C is beyond the maximum possible value.

Option B would produce a net force to the left along the incline!

## Example: Box on truck

A box with mass $m$ = 50 kg sits on the back of a truck. The coefficients of friction between the box and the truck are $\mu_K$ = 0.2 and $\mu_S$ = 0.4.

What is the maximum acceleration that the truck can have without the box slipping?

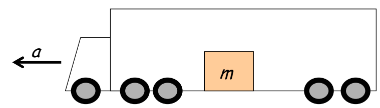
A. 2.0 m/s$^2$

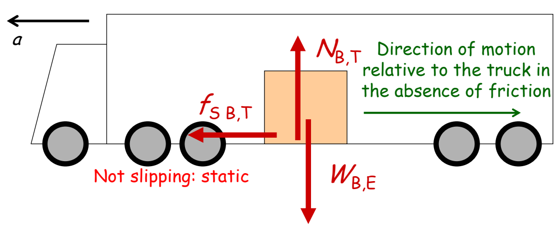
B. 3.1 m/s$^2$

C. 3.9 m/s$^2$

D. 4.9 m/s$^2$

E. 9.8 m/s$^2$




$f_S = m_B a \longrightarrow f_{S\,MAX} = m_B a_{MAX}$

$N - W = 0$

$$a_{\max} = \frac{f_{S,MAX}}{m_B} = \frac{\mu_S N}{m_B} = \frac{\mu_S m_B g}{m_B} = \mu_S g = 0.4g = 3.9 \text{ m/s}^2$$

Answer C

## Drag forces

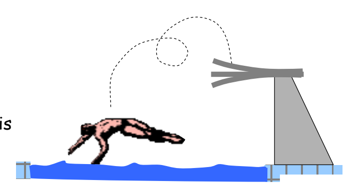
For solid-fluid relative motion, the friction force (called "drag force" or "resistance") depends on the relative speed:

$f_D = kv$ for low speeds

$f_D = Dv^2$ for high speeds

$k$ and $D$ depend on the geometry and the materials.

This is why this is going to hurt!



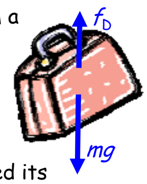
## Terminal speed

DEMOS: Parachutes / Marbles in corn syrup

Acceleration of a suitcase that falls from a plane: $mg - f_D = ma$

Increases with $v$

Eventually, $f_D = mg$, so $a = 0$!




When this happens, the system has reached its terminal speed:

$$mg - Dv_{\text{terminal}}^2 = 0$$

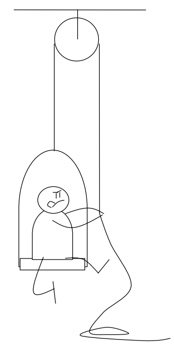
$$v_{\text{terminal}} = \sqrt{\frac{mg}{D}}$$

This is how parachutes work!

## EXAMPLE: Pulling yourself up

A kid with mass $m$ = 30 kg has designed a rough elevator to get to his tree-house. It's made of a seat of mass $M$ = 5 kg, a rope and a pulley. If you want to use the elevator, you sit on the seat and pull on the rope as shown below.

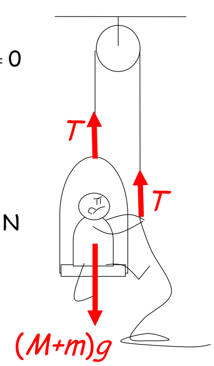
How strong is the kid pulling if the elevator is moving at constant speed?



$$2T - (m + M)g = (m + M)a = 0$$

$$T = \frac{m + M}{2} g$$

$$T = \frac{35 \text{ kg}}{2}(9.8 \text{ m/s}^2) = 172 \text{ N}$$



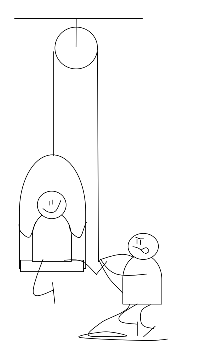

## ACT: Pulling somebody up

If, instead, a friend pulled with tension $T$ = 172 N on the loose end of the rope, the elevator would:

A. Move exactly as before.

B. Make it faster to the tree-house

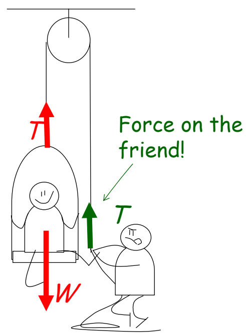
C. Not go up.



The net force on the elevator+kid system is now $F_{net} = T - W$

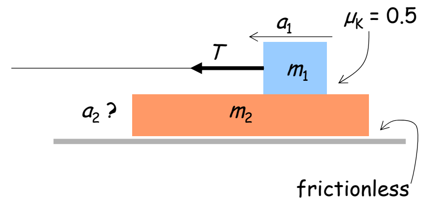
But $T = W/2$, so with this tension the net force on the system —and its acceleration – point down!

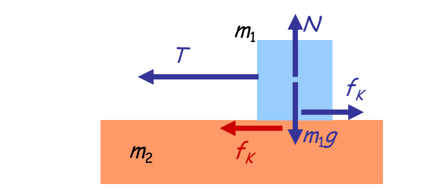
The friend needs to pull at least twice as hard!




## EXAMPLE: Box on another box

A box of mass $m_1$ = 1.5 kg is being pulled by a horizontal string with tension $T$ = 45 N. It **slides** with friction ($\mu_K$ = 0.50, $\mu_S$ = 0.70) on top of a second box of mass $m_2$ = 3.0 kg, which in turn sits on a frictionless floor. Find the acceleration of box 2.




For box 2: $f_K = \mu_K N = m_2 a_2$

From box 1, we know that $N - m_1 g = 0$

$$\longrightarrow \quad a_2 = \frac{\mu_k N}{m_2} = \mu_k \frac{m_1}{m_2} g = 0.50 \frac{1.5 \text{ kg}}{3.0 \text{ kg}} (9.8 \text{ m/s}^2) = 2.5 \text{ m/s}^2$$

**The magnitude of the tension did not play any role!**

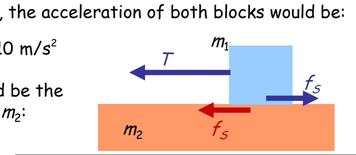
The tension just needs to be large enough so the boxes cannot move together.

If they moved together, the acceleration of both blocks would be:

$$a = \frac{T}{m_1 + m_2} = \frac{45 \text{ N}}{4.5 \text{ kg}} = 10 \text{ m/s}^2$$

The static friction would be the only horizontal force on $m_2$:

$$m_2 a = f_S$$




But static friction has a maximum value: $f_S \le \mu_S N = \mu_S m_1 g$

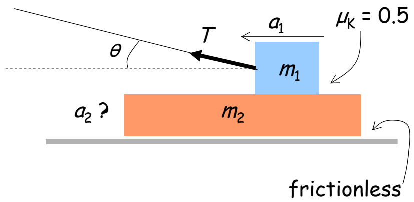
This imposes a lower limit to the coefficient of static friction: $m_2 a \le \mu_S m_1 g$

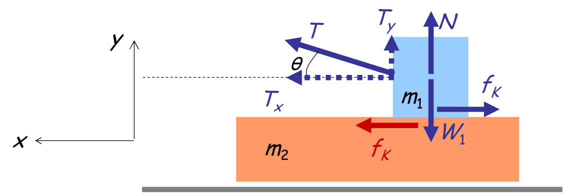
$$\mu_S \ge \frac{m_2 a}{m_1 g} = \frac{m_2 T}{(m_1 + m_2) m_1 g} = \frac{(3.0 \text{ kg})(45 \text{ N})}{(4.5 \text{ kg})(1.5 \text{ kg})(9.8 \text{ m/s}^2)} = 2.0 > 0.7$$

Block will not move together for this tension

## EXAMPLE: Box on another box (2)

Same problem ($m_1$ = 1.5 kg, $T$ = 45 N, $\mu_K$ = 0.50, $m_2$ = 3.0 kg), but now the string makes an angle $\theta$ = 15° with the horizontal. Find the acceleration of box 2.




For box 2: $f_K = \mu_K N = m_2 a_2$

From box 1, $N - m_1 g + T\sin\theta = 0$  $N < m_1 g$

$$\longrightarrow a_2 = \frac{\mu_k N}{m_2} = \mu_k \frac{m_1 g - T\sin\theta}{m_2}$$

$$= 0.50\frac{(1.5\text{ kg})(9.8\text{ m/s}^2) - 45\sin 15}{3.0\text{ kg}} = 0.51\text{ m/s}^2$$

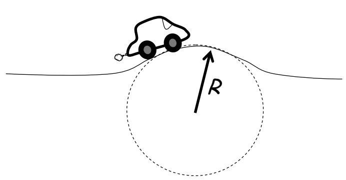
## ACT: Car on a bump

The pavement on Grand Ave. is higher along the center of the street than along the sides. So when you drive along 13th street and across Grand Ave., your car goes over a small hill. We can estimate the bump to have a radius of curvature of 30 m. What is the maximum speed your car should have if your wheels are to stay in contact with the ground all the time?

A. 12 m/s

B. 17 m/s

C. 20 m/s



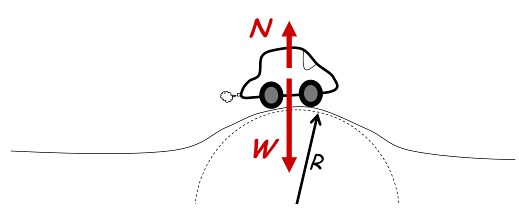

$W - N = ma_y = m\frac{v^2}{R}$

Large $v$ -> Small $N$

Small $v$ -> Large $N$

Smallest $N$: $N$=0 $\longrightarrow$ $mg = m\frac{v_{MAX}^2}{R}$

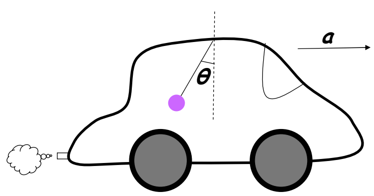
$v_{MAX} = \sqrt{Rg}$ = 17.1 m/s ≈ 60 mi/h



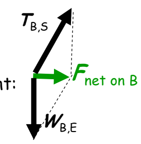
## EXAMPLE: Accelerometer

A car has a constant acceleration of 2 m/s$^2$. A small ball of mass $m$ = 0.5 kg attached to a string hangs from the ceiling.

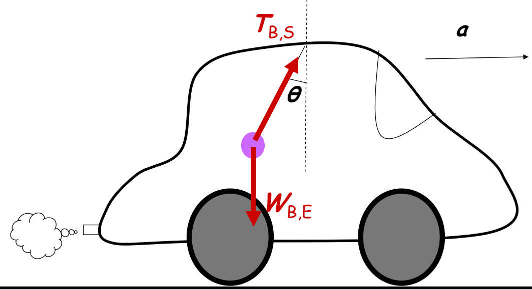
Find the angle $\theta$ between the string and the vertical direction.




Free-body diagram:



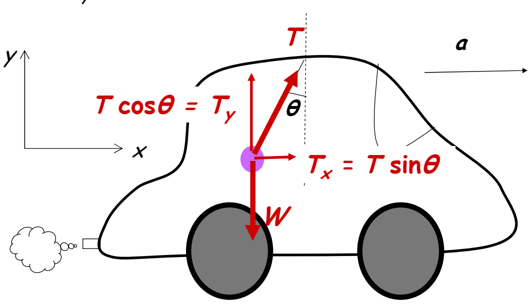

Net force must point to the right:




In components:

$x$: $T_x = ma$ $\qquad$ $T\sin\theta = ma$

$y$: $T_y - W = 0$ $\qquad$ $T\cos\theta - mg = 0$




2 equations, $\qquad T\sin\theta = ma$

2 unknowns $\qquad T\cos\theta - mg = 0$

$$\tan\theta = \frac{a}{g} \qquad T = \frac{mg}{\cos\theta} = m\sqrt{a^2 + g^2}$$

For $a$ = 2 m/s$^2$ , $\theta$ = 12°

Note: $\theta$ does not depend on the mass of the ball!

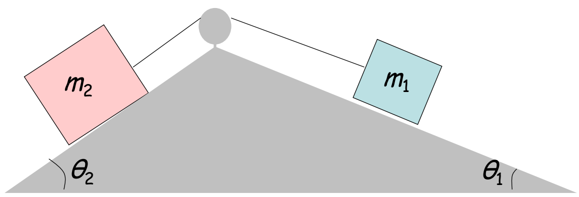
Check: For $a$ = 0 (constant speed), $\theta$ = 0°

This system is called an accelerometer.

It is also the principle behind windshield Dancing Elvis…

# EXAMPLE: Double incline

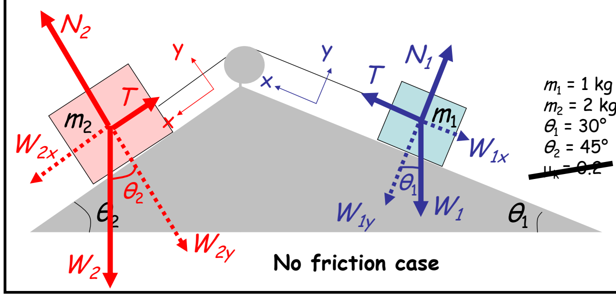
A box of mass $m_1$ = 1 kg sitting on a double incline is attached to another box of mass $m_2$ = 2 kg sitting on the other side of the incline by an ideal string that goes through an ideal pulley. The angles between the inclines and the horizontal are $\theta_1$ = 30° and $\theta_2$ = 45°. If the blocks are moving to the left and $\mu_k$ = 0.2, what is the acceleration of the system?




1: $T - W_{1x} = m_1 a$

2: $W_{2x} - T = m_2 a$

$$a = g\frac{m_2\sin\theta_2 - m_1\sin\theta_1}{m_1 + m_2} = 0.31\,g = 3.0\ \text{m/s}^2$$




1: $T - W_{1x} - f_{K1} = m_1 a$ where $f_{K1} = \mu_k N_1 = \mu_k m_1 g \cos\theta_1$

2: $W_{2x} - T - f_{K2} = m_2 a$ where $f_{K2} = \mu_k N_2 = \mu_k m_2 g \cos\theta_2$

$$a = g\frac{m_2(\sin\theta_2 - \mu_k\cos\theta_2) - m_1(\sin\theta_1 + \mu_k\cos\theta_1)}{m_1 + m_2} = 0.15\,g = 1.5\ \text{m/s}^2$$