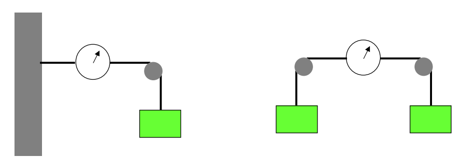
Lecture 11

Friction.
Dynamics of Circular Motion

ACT: Scale

A 40N block is hung from a rope attached to a scale. The scale is then attached to a wall and reads 40N. What will the scale read when it is instead attached to another 40N block, as shown in the second figure?



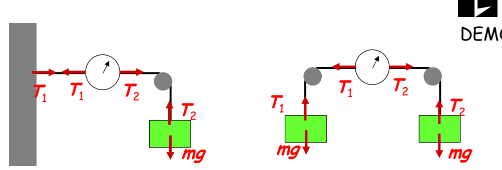
A. 0 N   B. 40N   C. 80N

The block has $a = 0$ — $T_2 = mg$

String 2 is massless (and has $a = 0$) — T_2 is the same on both sides

The scale has $a = 0$ — $T_1 = T_2$

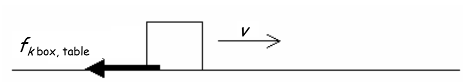
DEMO



The forces on the scale are exactly the same!

Friction

Friction always opposes relative motion.



It is caused by “microscopic” interactions between two “surfaces”.

Solid-solid → Surface friction (kinetic or static)

Solid-fluid → Drag forces

Fluid-fluid → Viscosity

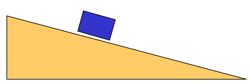
Surface friction (solid-solid)

Kinetic friction: When the relative motion between the two objects **is not** zero. It "slows down" the sliding motion.

Example: A box sliding on a horizontal table will eventually stop.

Static friction: When the relative motion between the two objects **is** zero. It prevents the sliding motion from happening.

Example: A box on an incline that doesn't slide down.



Kinetic friction

When the relative motion between the two objects is not zero. It "slows down" the sliding motion.

Experimentally, it is observed that the kinetic friction force between two surfaces:

- Is parallel to the surface (and thus perpendicular to the normal)
- Its magnitude does not depend on the speed (except when $v = 0$) or on the area of the contact surfaces.
- Its magnitude is proportional to the magnitude of the normal force between the two surfaces:

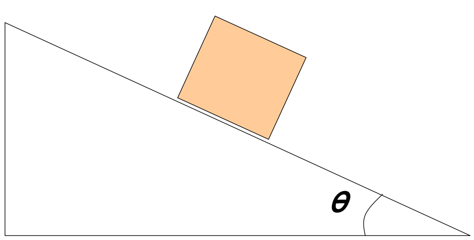
Use the correct one!!! (N is **NOT** always mg)

$$f_k = \mu_k N$$

μ_k = coefficient of kinetic friction (depends on the materials)

EXAMPLE: Box on incline with friction

A box slides down an incline with angle $\theta = 45°$. The coefficient of kinetic friction between the box and the plane is $\mu_k = 0.2$. Find the acceleration of the box.



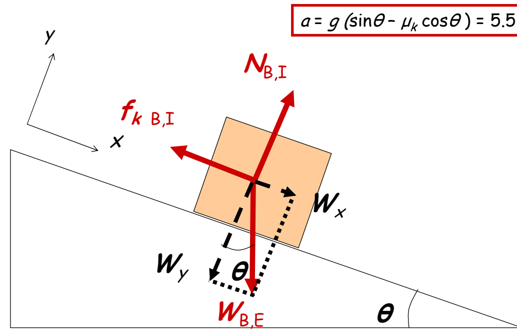
x: $W_x - f_k = ma$ → $mg\sin\theta - \mu_k N = ma$

y: $N - W_y = 0$ → $N - mg\cos\theta = 0$

→ $mg\sin\theta - \mu_k mg\cos\theta = ma$

↓

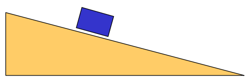
$$a = g(\sin\theta - \mu_k\cos\theta) = 5.5\ \text{m/s}^2$$



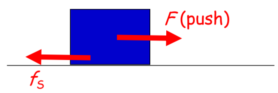
Static Friction

When the relative motion between the two objects is zero. It prevents the sliding motion from happening.

Example: A box on an incline that doesn't slide down.



Example: Pushing on a heavy box that does not move…



…unless you push hard enough

f_S has a maximum value (then the pushing force "wins")

Experimental facts about static friction

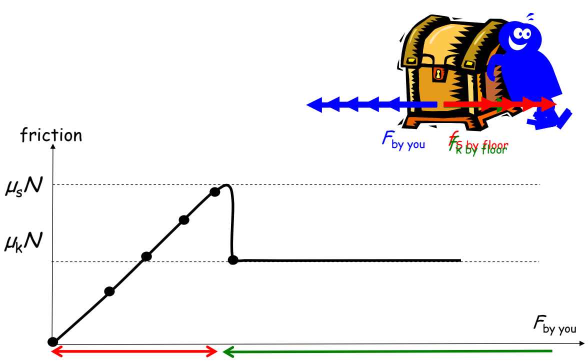
- It is found that $f_{S\ MAX} = \mu_S N$, so:

$$f_S \le \mu_S N$$

μ_s = coefficient of static friction (depends on the materials)

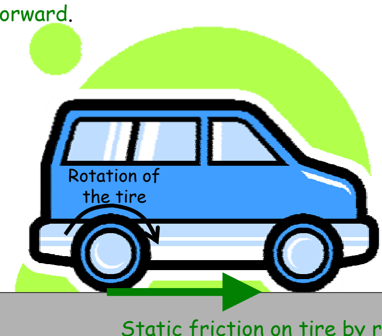
- Its magnitude does not depend on the area of the contact surfaces.
- $\mu_S \ge \mu_k$ (Otherwise things would not keep moving!)

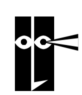
Example: Trying to move a trunk



Car

The static friction between the tire and the road pushes the car forward.



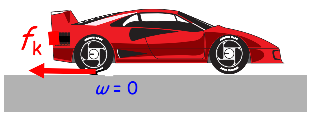


DEMO: Tire

Braking

When you brake normally, the kinetic friction between the brake pads and the rotors (disks that turn with the tires) slow you down.

If you block the tires (no ABS!), you skid on the road. The kinetic friction between the road and the tires slows you down.
This is less effective (pads and disks are designed to have a huge μ_k and the normal force between them is also very large) —and very bad for your tires…



ACT: Book against wall

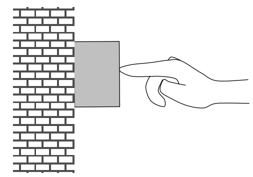
When you push a book against a wall, the static friction between the wall and the book can prevent it from falling.

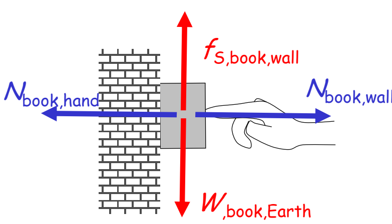
If you press harder, the friction force will be:

A. Larger than before

B. The same

C. Smaller than before.





For the book not to fall down, $f_S = W$

Pushing harder (increasing $N_{\text{book,hand}}$) increases $N_{\text{book,wall}}$ and therefore $f_{S\,MAX}$ increases, but not the actual value of f_S that we had, which needs to continue to be exactly W!

Dynamics of circular motion

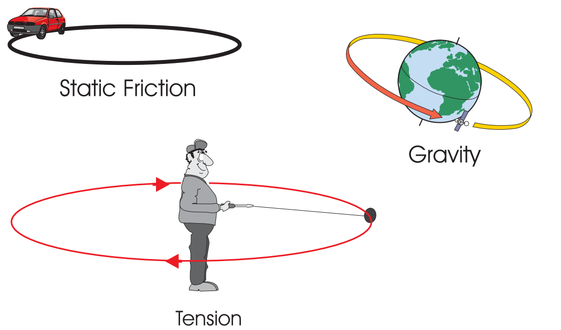
Conceptually, there's nothing new. All we need is an appropriate net force to produce the appropriate acceleration.

To have circular motion, we always need the centripetal acceleration:
- directed toward the center
- with magnitude $a = \frac{v^2}{r}$

We will always need a force:
- directed toward the center
- with magnitude $F = m\frac{v^2}{r}$ (called "centripetal force")

Be careful: This is not some *additional* force!

Who exerts the centripetal force?



EXAMPLE: Merry-go-round

Little Jacob (15 kg) sits on the edge of a merry-go-round of radius 1.0 m while big sister makes it turn… faster and faster. How fast can the system go before Jacob takes off if the coefficient of static friction between Jacob's pants and the merry-go-round is 0.5?

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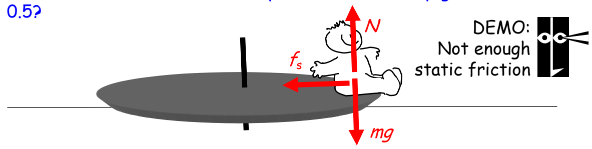


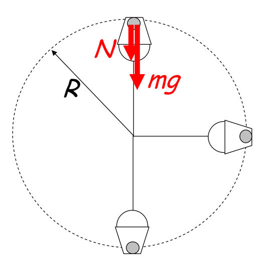
Static friction provides the needed radial acceleration: $f_S = mr\omega^2$

Maximum speed ↔ Maximum static friction: $\mu_S mg = mr\omega_{MAX}^2$

$$\omega_{MAX} = \sqrt{\frac{\mu_S g}{r}} = \sqrt{\frac{(0.5)(9.8\ \text{m/s}^2)}{1\ \text{m}}} = 2.2\ \text{rad/s} \approx \frac{1}{3}\ \text{turn/s}$$

Example: Bucket

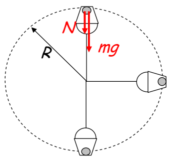
A stone of mass *m* sits at the bottom of a bucket. A string is attached to the bucket and the whole thing is made to move in circles. What is the minimum speed that the bucket needs to have at the highest point of the trajectory in order to keep the stone inside the bucket?



$$mg + N = ma$$

a needs to be $a = \frac{v^2}{R}$

$$mg + N = m\frac{v^2}{R}$$



$$mg + N = m\frac{v^2}{R}$$

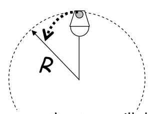
DEMO:
Bucket with water

- If v increases, N needs to be larger (if v becomes too large, since N is also the force on the bucket by the stone, the bottom of the bucket might end up broken…)

- If v decreases, N needs to be smaller. But at some point, N will become zero! This is the condition for the <u>minimum speed</u>:

$$mg = m\frac{v_{\text{min}}^2}{R} \quad \Rightarrow \quad \boxed{v_{\text{min}} = \sqrt{gR}}$$

The speed cannot get any smaller or the trajectory will not be a circle anymore (because the remaining forces –mg-- will produce an acceleration that is too strong for a circle of radius R —at that speed)



If $v < v_{\text{min}}$, the stone will do something like this…