# Lecture 10

## Applying Newton's Laws

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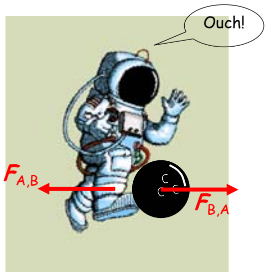
# ACT: Bowling on the Moon

An astronaut on Earth kicks a bowling ball horizontally and hurts his foot. A year later, the same astronaut kicks a bowling ball on the moon with the same force.

His foot hurts:

- A. More
- B. Less
- [✓ C. The same]



Movie from Apollo 17

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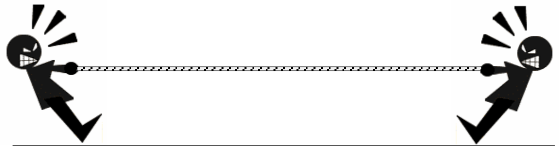
# "String Theory"

**Tension:** magnitude of the force acting across a cross-section of the rope/string/cable at a given position (it's the force you would measure if you cut the rope and grabbed the ends).



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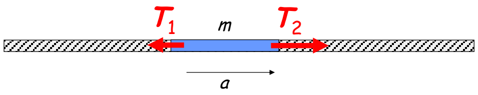
We'll assume **ideal** (constant length), **massless strings** (i.e, mass much smaller than the rest of the masses in the system).

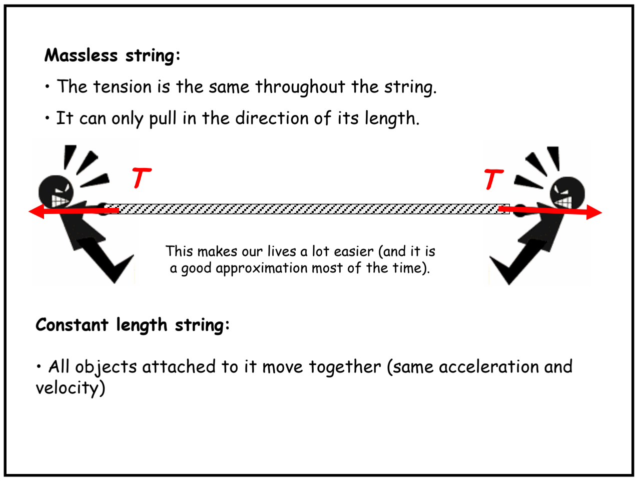
Consider a segment with mass $m$ of a rope with an acceleration $a$ to the right. If we neglect gravity, the forces on the segment are:



$$T_2 - T_1 = ma \quad \Rightarrow \quad T_2 > T_1$$

$$\text{If } m = 0, \quad T_2 - T_1 = 0 \quad \Rightarrow \quad T_2 = T_1$$

(also, then the weight of the segment really is negligible)



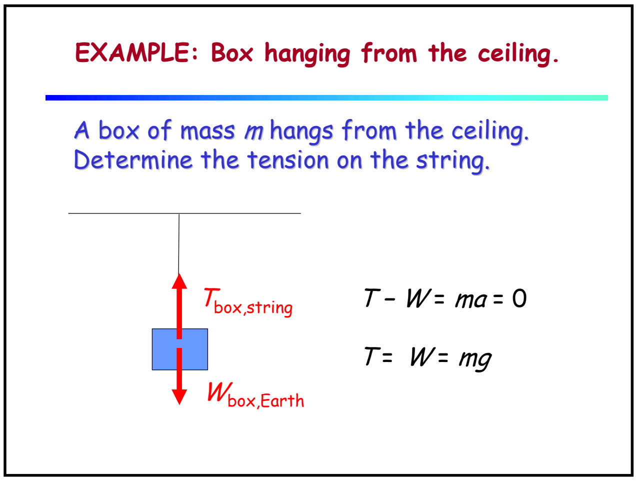
**Massless string:**

- The tension is the same throughout the string.
- It can only pull in the direction of its length.

This makes our lives a lot easier (and it is a good approximation most of the time).

**Constant length string:**

- All objects attached to it move together (same acceleration and velocity)

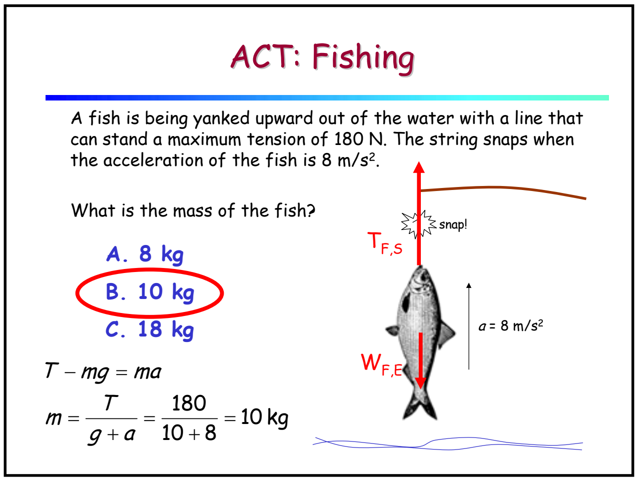


## EXAMPLE: Box hanging from the ceiling.

A box of mass $m$ hangs from the ceiling. Determine the tension on the string.

$T - W = ma = 0$

$T = W = mg$



## ACT: Fishing

A fish is being yanked upward out of the water with a line that can stand a maximum tension of 180 N. The string snaps when the acceleration of the fish is 8 m/s$^2$.

What is the mass of the fish?

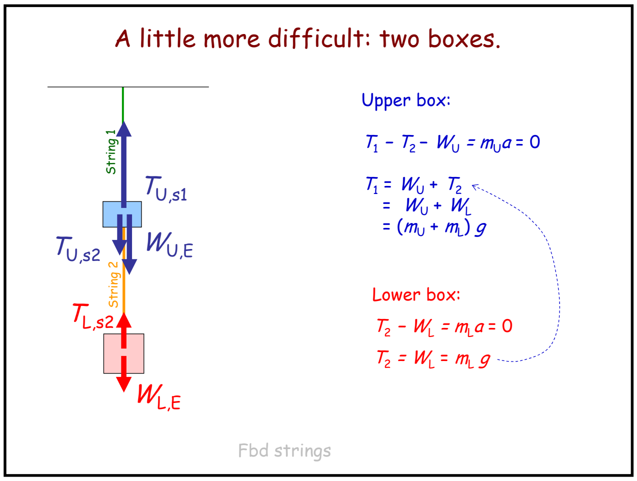
**A. 8 kg**

**B. 10 kg**

**C. 18 kg**

$$T - mg = ma$$

$$m = \frac{T}{g+a} = \frac{180}{10+8} = 10 \text{ kg}$$



## A little more difficult: two boxes.

Upper box:

$T_1 - T_2 - W_U = m_U a = 0$

$T_1 = W_U + T_2$
$= W_U + W_L$
$= (m_U + m_L)\ g$

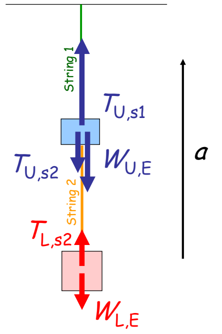
Lower box:

$T_2 - W_L = m_L a = 0$

$T_2 = W_L = m_L\ g$

Fbd strings

## What if the boxes hang from the ceiling of an accelerated elevator?



Upper box:

$T_1 - T_2 - W_U = m_U a$

Simply keep the $a$ in the equations!

Lower box:

$T_2 - W_L = m_L a$

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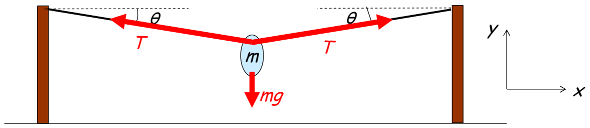
## Example: Cable

DEMO: Sharing the weight

Find the tension in the cables.



$$(x: \quad T\cos\theta - T\cos\theta = 0)$$

$$y: \quad 2T\sin\theta - mg = 0 \longrightarrow T = \frac{mg}{2\sin\theta}$$

Small $\theta$, large $T$

It is impossible for a real cable ($m > 0$) to be completely horizontal (it would require infinite tension, and then the cable snaps).

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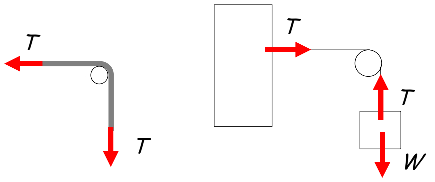
## Pegs and pulleys

Used to change the direction of forces.

Ideal massless pulley or ideal smooth peg: changes the direction of the force without changing its magnitude.



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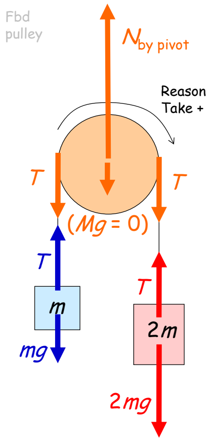
## Atwood's Machine



Reasonable direction of motion:
Take + in this direction

🟥 $2mg - T = 2ma$

🟦 $T - mg = ma \quad \Rightarrow \quad T = m(a+g)$

$$2mg - m(a+g) = 2ma$$

$$\boxed{a = \frac{g}{3} \quad ; \quad T = \frac{4}{3}mg}$$

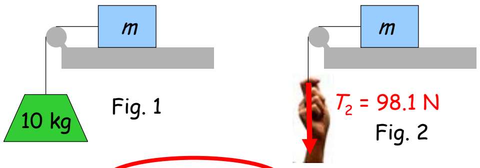
Note that: $mg < T < 2mg$
It's what I need to get the net force in the right direction to each box!

## ACT: A weight vs. a hand

In figure 1, a 10-kg mass hangs from a string and pulls on a box of mass *m*.
In figure 2, a hand provides a constant downward force of 98.1 N and pulls on another box of mass *m*.

The pulleys and strings are all ideal and massless.

Where does the box experience a larger acceleration?



A. In figure 1. [✓ B. In figure 2.] C. It's the same in both.

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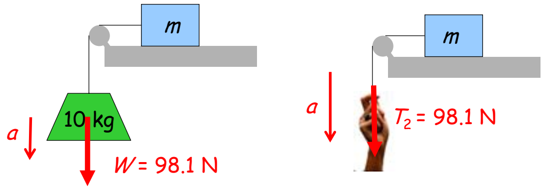
**Consider the whole system:**

| Figure 1: | Figure 2: |
|---|---|
| Net force: 98.1-N | Net force: 98.1-N |
| Total mass: 10 kg + *m* | Total mass: *m* |



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**Or consider the net force on the box:**

Figure 1:

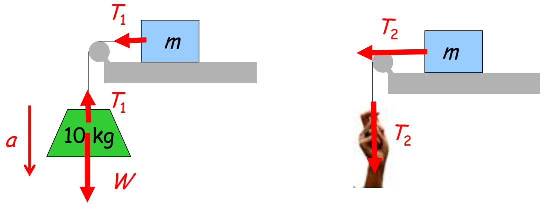
10-kg weight:

$W - T_1 = m_w a > 0$

So: $T_1 < W$  $\boxed{T_1 < 98.1\text{ N}}$

Figure 2:

$\boxed{T_2 = 98.1\text{ N}}$



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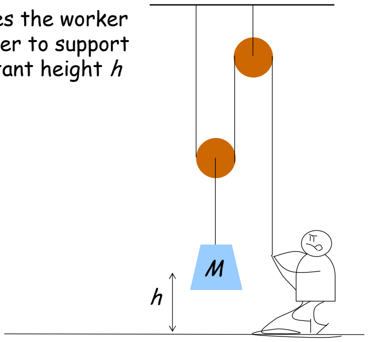
## Example: Pulley

How much force does the worker have to exert in order to support the mass *M* at constant height *h* off the ground?

A. $Mg$
[✓ B. $Mg/2$]
C. $Mgh$
D. $Mg/h$
E. $Mg/(2h)$



orange pulley: $T_2 - 2T_1 = 0$

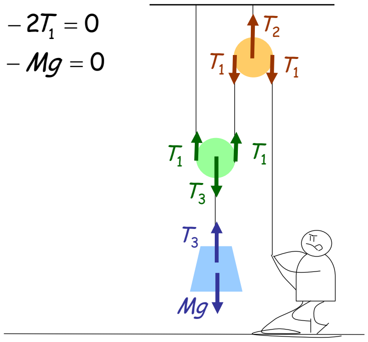
green pulley: $T_3 - 2T_1 = 0$

mass $M$: $T_3 - Mg = 0$

$$T_2 = T_3 = Mg = 2T_1$$

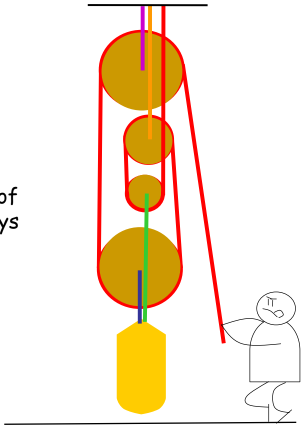
$$\Downarrow$$

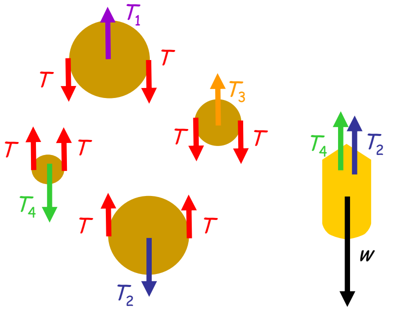
$$T_1 = \frac{Mg}{2}$$



## Example: Pulleys

A sack of weight $w$ hangs motionless from a system of pulleys. All ropes and pulleys are massless. What is the magnitude of the force is exerted by the worker?

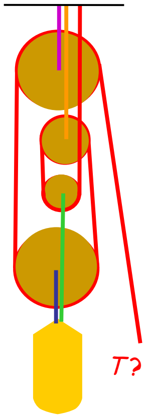




$T_1 = T_2 = T_3 = T_4 = 2T$

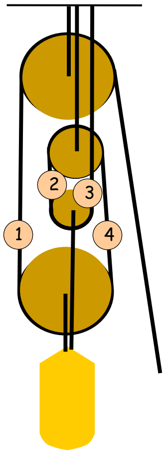
$T_2 + T_4 = w$

$w = 4T \longrightarrow \boxed{T = \dfrac{w}{4}}$



In practice, just count the number of ropes providing support.

$\longrightarrow \boxed{T = \dfrac{w}{4}}$

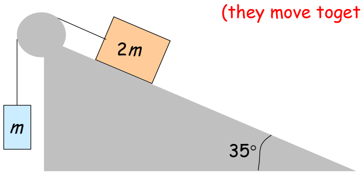


DEMO: Pulleys

# Going 2D: Incline and pulley

Find the acceleration of the boxes when the system below is released. Friction is negligible.

Same acceleration for both (they move together)



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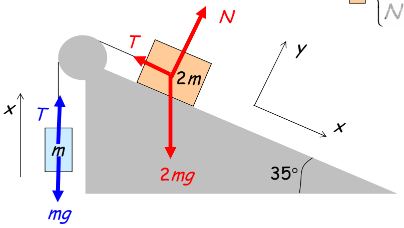
1. Draw free body diagram for both boxes.
2. Select axes
3. Write Newton's 2nd law

☐ $T - mg = ma$

▨ $\begin{cases} 2mg\sin\theta - T = 2ma \\ N - 2mg\cos\theta = 0 \end{cases}$



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4. Solve equations

$$\begin{cases} T - mg = ma \\ 2mg\sin\theta - T = 2ma \end{cases} \qquad T = m(g+a)$$

$$2mg\sin\theta - m(g+a) = 2ma$$

$$2g\sin\theta - g - a = 2a$$

$$\boxed{a = \frac{g}{3}(2\sin\theta - 1)} = \frac{g}{3}(2\sin 35^\circ - 1) = 0.049g = 0.48\ \text{m/s}^2$$

If $\theta < 30^\circ$, $a < 0$