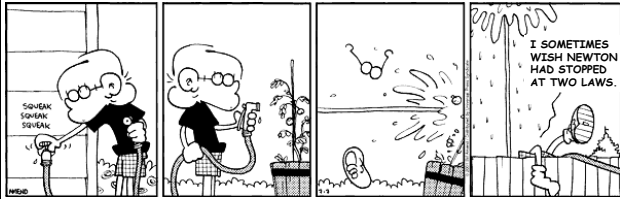


## Lecture 9 : Newton's Third and Free Body Diagrams



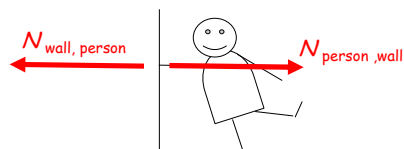
## Newton's Third Law

For every force, or action, there is an equal but opposite force, or reaction.

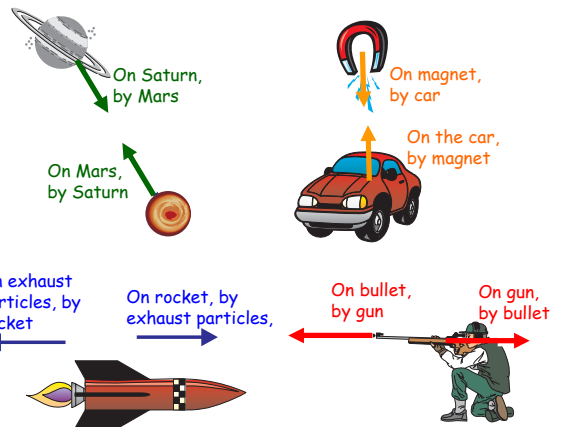
Forces ALWAYS happen in pairs.

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

## EXAMPLES

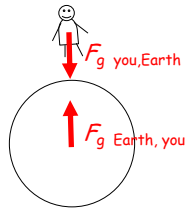


Person leaning against the wall



### Gravitation: You attract the Earth!

$$F_{g, \text{you, Earth}} = F_{g, \text{Earth, you}}$$

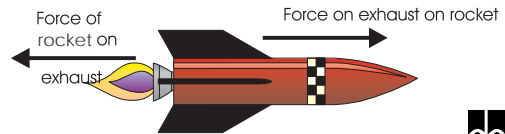


But the acceleration that this produces on the Earth is

$$a = \frac{\text{your weight}}{M_{\text{Earth}}} \approx \frac{(70 \text{ kg})(10 \text{ m/s}^2)}{6 \times 10^{24} \text{ kg}} \approx 10^{-22} \text{ m/s}^2,$$

nothing to be too proud of...)

Newton's third law implies that if a rocket accelerates forwards, something must be pushed backwards. In outer space, there isn't much else around besides its own fuel.



DEMO: What if roads where not properly fixed to the ground?

### ACT: Carts with spring

Two carts are put back-to-back on a track. Cart A has a spring-loaded piston; cart B, which has twice the mass of cart A, is entirely passive. When the piston is released, it pushes against cart B, and the carts move apart.

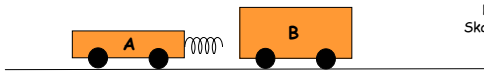
Which of the two forces exerted by the two carts on each other has a larger magnitude?

1. The force exerted by A
2. The two forces have equal magnitude.
3. The force exerted by B.

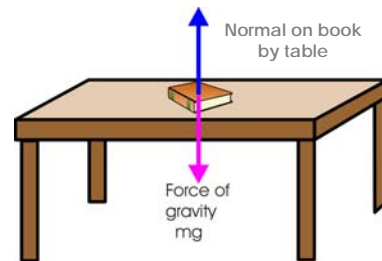
It's a third law pair!!

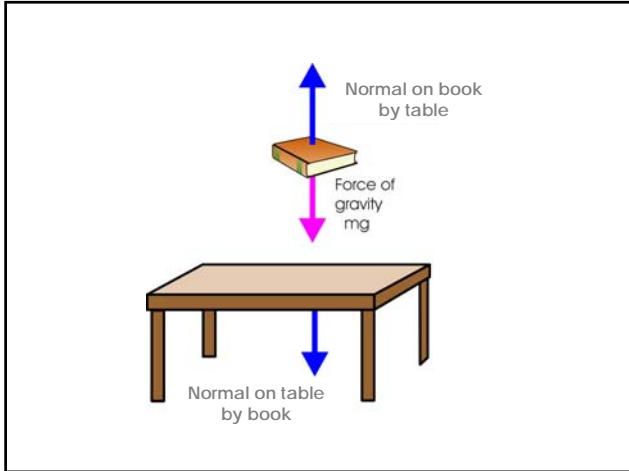


DEMO: Skateboards



### A Book on a Table





### Book on Table - The full story

**Action-Reaction Pairs**

Normal force between book and table  
 $N_{BT} = -N_{TB}$

Gravitational force between book and earth  
 $W_{BE} = -W_{EB}$

Normal force between table and earth  
 $N_{TE} = -N_{ET}$

Gravitational force between table and earth  
 $W_{TE} = -W_{ET}$

The book does not accelerate  $W_{BE} + N_{BT} = 0$   
 The table does not accelerate  $W_{TE} + N_{TB} + N_{TE} = 0$   
 Does the earth accelerate?

### Setting limits to Newton's Laws

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Galilean transformations for accelerations:  $\vec{a}_{p,A} = \vec{a}_{p,B} + \vec{a}_{B,A}$

When system B is accelerated in relation to A, funny things happen...

Imagine an object moving in a straight line at constant speed relative to B ( $\vec{a}_{p,B} = 0$ ). If B is accelerated relative to A, the object will appear to have a non-zero acceleration from the point of view of A!  
 ...and this could result in a curved trajectory!!

Tricky puck on air table.

Other examples:  
 Standing in a bus that brakes sharply (passenger "falls forward").  
 Acceleration simulator (astronaut feels "pushed against the seat")

### Inertial and Non-inertial frames of reference

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Inertial frame of reference: moves at constant velocity relative to the fixed stars (Mach's Principle: "funny things" don't happen → Newton's laws hold)

Non-inertial frame of reference: is accelerated with respect to an inertial frame of reference (and "funny things" happen → Newton's laws don't hold).

They can be very tricky!!

DEMO:  
Passenger  
on "bus"

## Where do Newton's Laws Work?

Newton's laws are true in Inertial Reference Frames (IRF).

In a non-inertial ref. frame, you can have an acceleration without having a force → we think there's a force (we're applying the 2<sup>nd</sup> law!). These are "fictitious" forces (the most popular one: the "centrifugal" force).

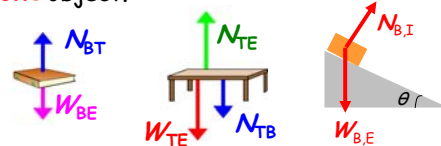
Is Ames a good IRF?

$$a_{\text{Ames}} = R\omega^2 = R\left(\frac{2\pi}{T}\right)^2 \rightarrow a_{\text{Ames}} \approx 0.034 \text{ m/s}^2 \ll 9.8 \text{ m/s}^2$$

where  $T = 1 \text{ day} = 8.64 \times 10^4 \text{ s}$   
 $R_{\text{Earth}} \approx 6.4 \times 10^6 \text{ m}$   
 (pretty decent IRF)

## Free Body Diagram

It is a diagram with **all** the forces acting on **one** object.



You should always draw a free-body diagram before attempting an application of Newton's second law!!! \*

\* This instructor declines all responsibility for a failed question and will disregard any whining if a free-body diagram has not been drawn.

## Example: Apparent weight

John has a mass of 100 kg and standing on a scale in an elevator which is accelerating upwards from rest at  $2 \text{ m/s}^2$ . What will the scale read?

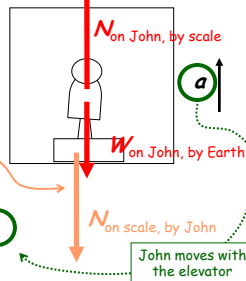
What does a scale measure?

The magnitude of the normal force on the scale by John,  $|N_{JS}| = |N_{SJ}|$

(not part of John's free body diagram)

Newton's 2<sup>nd</sup> law on John:

$$N_{JS} - W_{JE} = m_j a$$



John has a mass of 100 kg and standing on a scale in an elevator which is accelerating upwards from rest at  $2 \text{ m/s}^2$ . What will the scale read?

$$N_{JS} - W_{JE} = m_j a$$

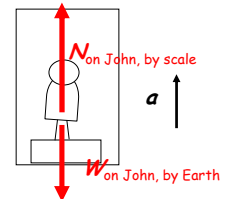
$$N_{JS} - m_j g = m_j a$$

$$N_{JS} = m_j g + m_j a$$

$$= m_j (g + a) =$$

$$= (100 \text{ kg})(9.8 \text{ m/s}^2 + 2.0 \text{ m/s}^2)$$

$$= 1180 \text{ N}$$



If the scale is in kg, it will read:  $\frac{N_{JS}}{g} = \frac{1180 \text{ N}}{9.8 \text{ m/s}^2} = 120 \text{ kg}$

**Check:** When the elevator is at rest ( $a = 0$ ), the scale must read the "correct" weight, 100 kg (980N).

Note: In Physics -or at least in this course-, the word "weight" refers to  $mg$ , not to what a scale reads (which we sometimes call "apparent weight" or "perceived weight").

### Back to Free Fall

If we neglect friction, only one force is acting:



Newton's second law:  $\vec{F}_{\text{net}} = m\vec{a}$

$$mg = ma$$

$$a = g = 9.81 \text{ m/s}^2$$

All falling bodies have the same acceleration of  $9.81 \text{ m/s}^2$  because:

- The  $m$  in  $mg$  and the  $m$  in Newton's second law are the same (Equivalence of Gravitational and Inertial Mass. This is the basis of General Relativity!).
- Weight is the only force acting!

NOTE: Weight will always be there -in problems near the Earth-, but most of the time, it is NOT the only force. So the acceleration will NOT be  $9.81 \text{ m/s}^2$ .

### ACT: Force and acceleration (2)

Two blocks of masses  $m$  and  $2m$  are pushed together along a horizontal, frictionless surface by a force  $F$ . The magnitude of the net force on block B is:

- A.  $1/3 F$
- B.  $2/3 F$**
- C.  $F$



Entire system:  $F_{\text{net,all}} (= F) = 3ma$  (so  $ma = \frac{F}{3}$ )

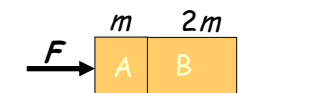
Block B:  $F_{\text{net,B}} = 2ma = 2 \frac{F}{3}$

Block A:  $F_{\text{net,A}} = ma = \frac{F}{3}$

### ACT: Force and acceleration (3)

Two blocks of masses  $m$  and  $2m$  are pushed together along a horizontal, frictionless surface by a force  $F$ . The magnitude of the force on block A by block B is:

- A.  $1/3 F$
- B.  $2/3 F$**
- C.  $F$



Action-reaction pair  
 $N_{\text{on A by B}} = N_{\text{on B by A}} \equiv N$

Block A:      x:  $F - N = ma$       (y:  $mg - N_A = 0$ )  
 Block B:      x:  $N = 2ma$       (y:  $2mg - N_B = 0$ )

We can use what we already know:  $N = F_{\text{net,B}} = \frac{2}{3}F$

Or solve the system:  
 $N = 2(F - N) \Rightarrow N = \frac{2}{3}F$

### Example: Box on an incline

A hand keeps a 35-kg box from sliding down a frictionless incline. The plane of the incline makes an angle  $\theta = 25^\circ$  with the horizontal. What is the magnitude of the force exerted by hand?

- Draw the free-body diagram
- Choose axes (draw them!)
- Use Newton's 2<sup>nd</sup> law in the x and y-directions.

A. 35 N  
 B. 311 N  
 C. 343 N  
 D. 145 N  
 E. 100 N

A hand keeps a 35-kg box from sliding down a frictionless incline. The plane of the incline makes an angle  $\theta = 25^\circ$  with the horizontal. What is the magnitude of the force exerted by hand?

$$mg \sin \theta - F = 0 \quad (a_x = 0)$$

$$N - mg \cos \theta = 0 \quad (a_y = 0)$$

$$F = mg \sin \theta$$

$$= (35 \text{ kg})(9.8 \text{ m/s}^2) \sin(25^\circ)$$

$$= \boxed{145 \text{ N}} \quad (\text{Answer D})$$