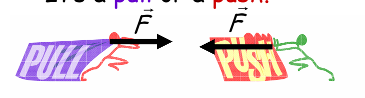
Lecture 8

Forces: Newton's First and Second Laws

What is a force?

It's a pull or a push.



It has a magnitude and a direction

↓

Forces are vectors!

Types of forces

1. Contact forces:

- Normal force (always perpendicular to contact surface); it's the basic "push"
- Tension (force through a string, cable, rope, etc): it's the basic "pull"
- Friction (appears when two surfaces slide or try to slide one against the other)

2. Action at distance or non-contact forces:
(The fundamental forces of Nature!)

- Gravitational
- Electric and Magnetic
- Weak force (important in subatomic world)
- Strong force (important in subatomic world)

Newton's Three Laws

1. Any object remains at rest or in motion along a straight line with constant speed unless acted upon by a net force.

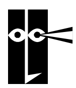
2. $\vec{F}_{net} = \sum_{all} \vec{F} = m\vec{a}$

3. Forces occur in pairs: $\vec{F}_{on\ B\ by\ A} = -\vec{F}_{on\ A\ by\ B}$
For every force (or action) there is an equal but opposite force (or reaction)

Newton's First Law

Any object remains at rest or in motion along a straight line with constant speed unless acted upon by a net force

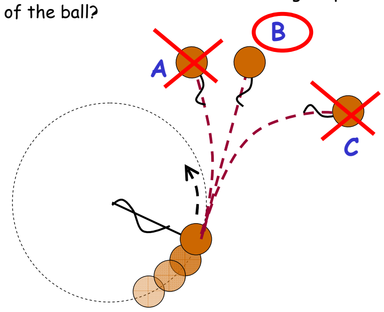
This law tells you what happens in the absence of any force, for instance in outer space



DEMO:
Air table

ACT: Snapped string

A small ball attached to the end of a string moves in horizontal circles on a frictionless table as shown below. If the string snaps, what will be the trajectory of the ball?



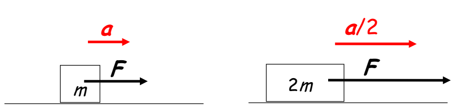
Newton's Second Law

The net force applied to an object is proportional to its acceleration: $\vec{F}_{net} = m\vec{a}$

Units: 1 N = 1 kg m/s^2

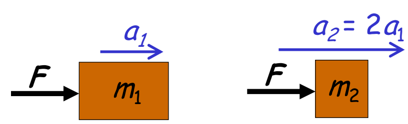
(1 lb = 4.448 N)

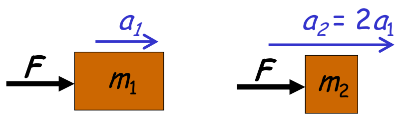
This is in fact the definition of mass!



ACT: Force and acceleration

A force F acting on a mass m_1 results in an acceleration a_1. The same force acting on a mass m_2 results in an acceleration a_2 = 2a_1.



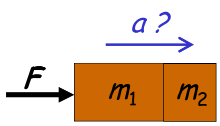


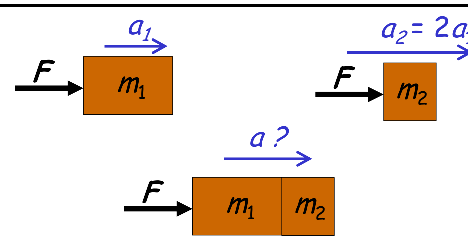
If both masses are put together and the same force is applied to the combination, what is the resulting acceleration?

A. $2/3\ a_1$

B. $3/2\ a_1$

C. $3/4\ a_1$



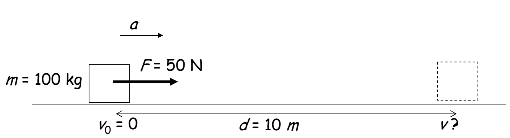


$$m_1 = \frac{F}{a_1} \qquad m_2 = \frac{F}{a_2} = \frac{F}{2a_1}$$

$$a = \frac{F}{m_1 + m_2} = \frac{F}{\frac{F}{a_1} + \frac{F}{2a_1}} = \frac{a_1}{1 + \frac{1}{2}} = \frac{2}{3}a_1$$

EXAMPLE: Pushing a box on ice.

A skater is pushing a heavy box (m = 100 kg) across a sheet of ice (horizontal and frictionless). He applies a horizontal force of 50 N on the box. If the box starts at rest, what is its speed v after being pushed over a distance d = 10 m?



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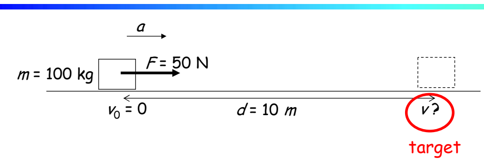


$$v^2 - v_0^2 = 2a\Delta x$$

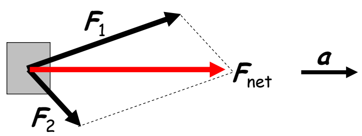
$$v = \sqrt{v_0^2 + 2a\Delta x} = \sqrt{0 + 2\frac{F}{m}\Delta x} = \sqrt{\frac{2Fd}{m}} = 3.2 \text{ m/s}$$

$$a = \frac{F_{net}}{m} = \frac{F}{m}$$

Remember: What is proportional to the acceleration is the NET force.

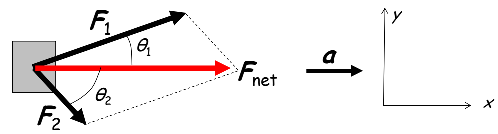
EXAMPLE: Two people are pulling on a box initially at rest with forces $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$. In which direction will the box move?

(top view)



The acceleration of the box points in the direction of $F_{net} = F_1 + F_2$.

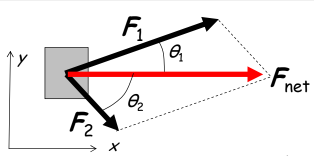
EXAMPLE: Two people are pulling on a box with forces $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$.



Let $\theta_1 = 20°$, $\theta_2 = 40°$, $m_{box} = 5.0$ kg and $a = 1.1$ m/s^2. What is the magnitude of the forces?

$$F_{net,x} = ma \qquad F_1\cos\theta_1 + F_2\cos\theta_2 = ma \qquad \text{2 equations}$$

$$F_{net,y} = 0 \qquad F_1\sin\theta_1 - F_2\sin\theta_2 = 0 \qquad \text{2 unknowns}$$



$\theta_1 = 20°$
$\theta_2 = 40°$
$m_{box} = 5.0$ kg
$a = 1.1$ m/s^2

$$F_1\sin\theta_1 - F_2\sin\theta_2 = 0 \longrightarrow F_1 = F_2\frac{\sin\theta_2}{\sin\theta_1}$$

$$F_1\cos\theta_1 + F_2\cos\theta_2 = ma$$

$$F_2\left(\frac{\sin\theta_2}{\sin\theta_1}\cos\theta_1 + \cos\theta_2\right) = ma$$

$$F_2 = \frac{ma}{\frac{\sin\theta_2}{\tan\theta_1} + \cos\theta_2} = \frac{(5.0\text{ kg})(1.1\text{ m/s}^2)}{\frac{\sin 40°}{\tan 20°} + \cos 40°} = \boxed{2.2\text{ N}}$$

$$F_1 = F_2\frac{\sin\theta_2}{\sin\theta_1} = \frac{ma}{\frac{\sin\theta_1}{\sin\theta_2}\left(\frac{\sin\theta_2}{\tan\theta_1} + \cos\theta_2\right)}$$

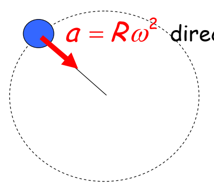
$$= \frac{ma}{\frac{\sin\theta_1}{\tan\theta_2} + \cos\theta_1} = \frac{(5.0\text{ kg})(1.1\text{ m/s}^2)}{\frac{\sin 20°}{\tan 40°} + \cos 20°} = \boxed{4.1\text{ N}}$$

Checks:

- Units
- Limits:
  - $\theta_1 = \theta_2$
  - $\theta_1 = \theta_2 = 0$
  - $m \to 0, \infty$
  - $a \to 0, \infty$

Example: Circular motion

A 2.0-kg ball at the end of a 1.5-m long string moves uniformly in horizontal circles. The force exerted by the string is 48 N. What is the speed of the ball?



$a = R\omega^2$ direction to the center of the circle

$$F = ma = mR\omega^2$$

$$\omega = \sqrt{\frac{F}{mR}}$$

$$= \sqrt{\frac{(48\text{ N})}{(2.0\text{ kg})(1.5\text{ m})}} = 4.0\text{ rad/s}$$

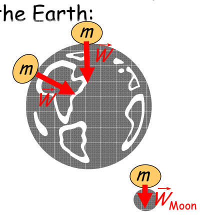
Weight

The weight of an object is the force of attraction of gravity by Earth on the object (usually near the Earth). It is observed that, near the surface of the Earth:

$$\vec{W} = m\vec{g}$$

$|\vec{g}| \sim 9.81$ m/s^2

direction: toward the center of the Earth

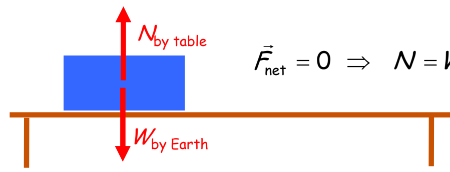


Weight and mass are not the same thing!!

The direction and magnitude of weight (vector) changes in different places. Mass (scalar) is always the same.
(On the Moon, for instance , g = 1.67 m/s^2)

EXAMPLE 1: Block on table

A block is placed on a horizontal surface. What forces are acting on the block?



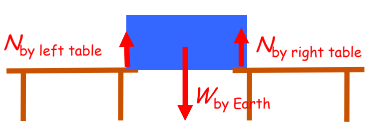
$$\vec{F}_{net} = 0 \Rightarrow N = W$$

If weight was the only force, there would be a net force on the box pointing down → an acceleration pointing down!

There has to be another force to achieve F_{net} = 0.

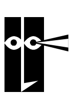
EXAMPLE 2: Block on two tables

A block is balanced in the space between two tables as shown below. What forces are acting on the block?



$$\vec{F}_{net} = 0$$
$$\Downarrow$$
$$N_R = N_L = \frac{W}{2}$$

($\frac{1}{2}$ with enough symmetry)



DEMO:
Nail bed

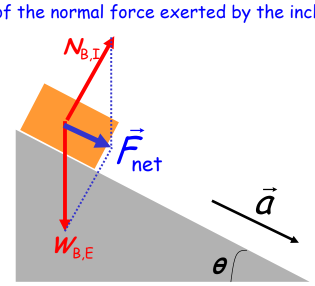
If the block rests on 100 mini-tables, each table exerts a relatively small force:

$$N_{\text{each table}} = \frac{W}{100}$$

Example: Box on an incline (I)

A box of mass m is released from rest on a frictionless incline that makes an angle θ with the horizontal. Find:

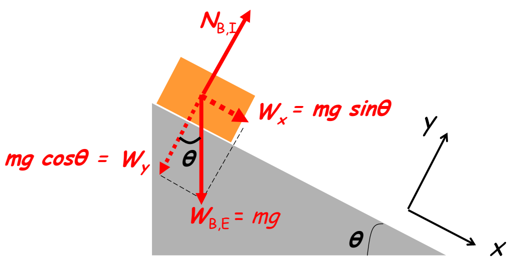
a. The acceleration of the block
b. The magnitude of the normal force exerted by the incline



$$x: \quad mg\sin\theta = ma_x$$
$$y: \quad N - mg\cos\theta = ma_y = 0$$

$$a_x = g\sin\theta$$
$$N = mg\cos\theta$$

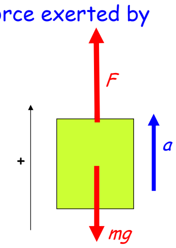
Not mg !!



EXAMPLE: Elevator moving up

A 200-kg elevator begins moving up with an acceleration of 3.0 m/s². Find the magnitude of the force exerted by the cable.

Draw a figure and select axes.

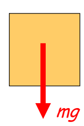


Newton's second law: $\vec{F}_{net} = m\vec{a}$

$$F - mg = ma$$
$$F = m(a+g) =$$
$$= (200\ \text{kg})(9.8\ \text{m/s}^2 + 3.0\ \text{m/s}^2) =$$
$$= 2560\ \text{N}$$

Checks: If a increases, F increases.
For a = 0, F = mg

Back to Free Fall



If we neglect friction, only one force is acting:

Newton's second law: $\vec{F}_{net} = m\vec{a}$

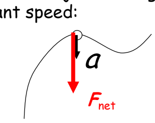
$$mg = ma$$
$$a = g = 9.81\ \text{m/s}^2$$

All falling bodies have the same acceleration of 9.81 m/s² because:

- The m in mg and the m in Newton's second law are the same (Equivalence of Gravitational and Inertial Mass. This is the basis of General Relativity!).
- Weight is the only force acting!

NOTE: Weight will always be there –in problems near the Earth-, but most of the time, it is NOT the only force. So the acceleration will NOT be 9.81 m/s².

Sometimes we know what the acceleration must look like. Imagine an object moving along the following trajectory at constant speed:



We know the acceleration must be like this (if speed is constant). What does the net force vector look like?

Example: Circular motion.
We know from lecture 6 what is the correct direction of acceleration in each of these cases. Many forces might be acting on the object. But we know the result of all these contributions must point in the direction of the acceleration.