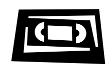
Lecture 7

Relative Motion

Goals for Lecture 7

- Define Galilean transformation laws between moving reference frames.
- Define inertial reference frames where Newton's laws live (even though we still don't know about Newton's laws…).



Upside down

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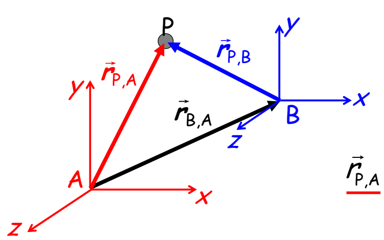
Galilean transformations

We always need to refer positions (and therefore velocities and accelerations) to a <u>frame of reference</u>.

= relation between the description of a particle in two frames which are moving with respect to each other.



$$\vec{r}_{P,A} = \vec{r}_{P,B} + \vec{r}_{B,A}$$

Galilean transformations

$$\vec{r}_{P,A} = \vec{r}_{P,B} + \vec{r}_{B,A}$$

$$\downarrow \frac{d}{dt}$$

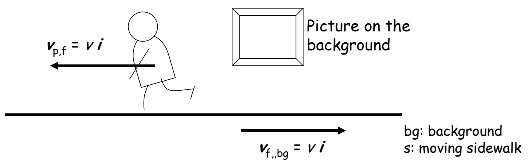
$$\vec{v}_{P,A} = \vec{v}_{P,B} + \vec{v}_{B,A}$$

$$\downarrow \frac{d}{dt}$$

$$\vec{a}_{P,A} = \vec{a}_{P,B} + \vec{a}_{B,A}$$

Example: Moving Sidewalk

A person walking on moving sidewalk: You can have $v_{person,background} = 0$ (not moving relative to a picture on the back wall):

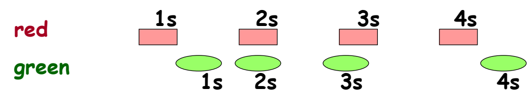


Clearly velocity is a reference-frame dependent quantity!

What are some frame independent quantities? Mass, time, temperature…

ACT: Relative motion of two cars

Two cars A and B move along parallel tracks. Shown are the snapshots of their motion at 1s intervals.

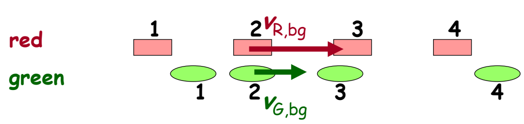


At $t = 2$ s, the velocity of the **green** car with respect to the **red** car:

A. Points to the right.

B. Points to the left.

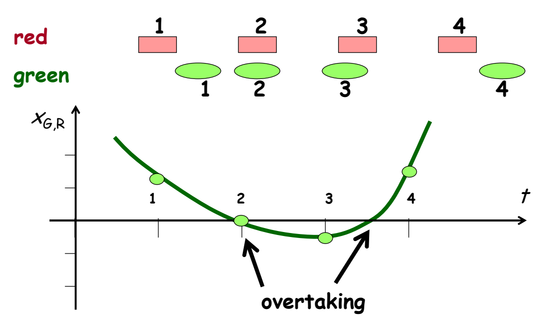
C. Is zero.



$$v_{G,bg} = v_{G,R} + v_{R,bg}$$

$$v_{G,R} = v_{G,bg} - v_{R,bg}$$

Position of **green** relative to **red**:



Example: Airport race

Two bored kids stuck at the airport (flight delays) decide to race. Both kids walk with speed v_w. One kid (A) will walk on the ground while the other (B) will walk on the "moving sidewalk" that moves with speed v_0. The race is roundtrip. Which kid wins the race?

A. Kid A

B. Kid B

C. Tie

D. Depends on the ratio v_w / v_0

E. Depends on the sign of v_0

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Time for roundtrip, kid A: $t_A = 2\frac{d}{v_w}$ Let d = length of the moving sidewalk.

Time for roundtrip, kid B: $t_B = t_{\text{against SW}} + t_{\text{with SW}}$

$v_{\text{kid B relative to ground}} = v_w - v_0$ $v_{\text{kid B relative to ground}} = v_w + v_0$

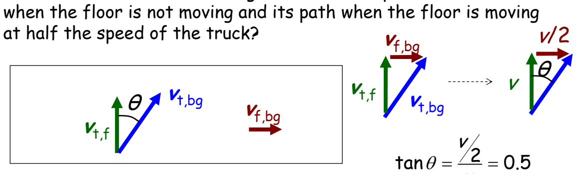
$$t_B = \frac{d}{v_w - v_0} + \frac{d}{v_w + v_0} = \frac{2v_w d}{v_w^2 - v_0^2} = \frac{2v_w d}{v_w^2}\left(\frac{1}{1-\frac{v_0^2}{v_w^2}}\right) = t_A\left(\frac{1}{1-\frac{v_0^2}{v_w^2}}\right)$$

If $v_0 < v_w$, then $\frac{1}{1-\frac{v_0^2}{v_w^2}} > 1$, so $t_A < t_B$ (answer A)

2D relative motion

Toy bulldozer on moving floor

From the film: What is the angle between the path of the truck when the floor is not moving and its path when the floor is moving at half the speed of the truck?



$$\tan\theta = \frac{v/2}{v} = 0.5$$

So the trajectory makes an angle $\theta = \tan^{-1}(0.5) = 26.6°$ with the vertical.

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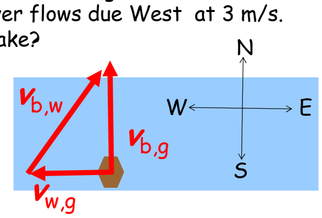
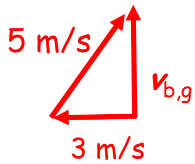
EXAMPLE: Boat

A boat whose engines can make it move at 5 m/s relative to the water is trying to go across a 100-m wide river to a point on the opposite shore and right North of its starting position. The river flows due West at 3 m/s. How long does the trip take?

$$v_{b,g} = v_{b,w} + v_{w,g}$$





$$v_{b,g} = \sqrt{(5\text{ m/s})^2 - (3\text{ m/s})^2} = 4\text{ m/s}$$

$$\Delta t = \frac{\Delta x_g}{v_{b,g}} = \frac{100\text{ m}}{4\text{ m/s}} = 25\text{ s}$$

Faster than light?

Two cars driving at 50 mph head toward each other on a highway. What is their relative speed?

Answer: 100 mph

Two spaceships moving at 200,000 km/s relative to Earth head toward each other somewhere in the galaxy . What is their relative speed?

Answer: 400 000 km/s

This is faster than light (300 000 km/s) !!
Is this possible???

No.

Special relativity (postulated by A. Einstein): Nothing can travel faster that light in vacuum ($c \sim 3 \times 10^8$ m/s) in <u>any</u> frame of reference.

The fix: Galilean transformations are OK for speeds $v \ll c$ only. For high speeds, we need to use the so-called Lorentz transformations (which become the Galilean transformation in the limit of small v).

Accelerated frames of reference

Galilean transformations for accelerations: $\vec{a}_{P,A} = \vec{a}_{P,B} + \vec{a}_{B,A}$

When system B is accelerated in relation to A, funny things happen…

Imagine an object moving in a straight line at constant speed relative to B ($a_{P,B} = 0$). If B is accelerated relative to A, the object will appear to have a non-zero acceleration from the point of view of A!

…and this could result in a curved trajectory!!


Tricky puck on air table.

Other examples:

Standing in a bus that brakes sharply (passenger "falls forward").

Acceleration simulator (astronaut feels "pushed against the seat")

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Inertial and Non-inertial frames of reference

<u>Inertial frame of reference</u>:

- moves at constant velocity relative to the fixed stars
- Mach's Principle: "funny things" don't happen → Newton's laws hold

<u>Non-inertial frame of reference</u>:

- is accelerated with respect to an inertial frame of reference
- "funny things" happen → Newton's laws don't hold.

They can be very tricky!!