Lecture 4

Constant acceleration in 1D

$x(t) \leftrightarrow v(t) \leftrightarrow a(t)$

In general, for 1D motion along a straight line:

$$a = \frac{dv}{dt} \quad \Leftrightarrow \quad v = v_0 + \int_0^t a(t)dt$$

$$v = \frac{dx}{dt} \quad \Leftrightarrow \quad x = x_0 + \int_0^t v(t)dt$$

One Dimensional Constant acceleration.

When a = constant, the equations are simple:

$$a = \frac{dv}{dt} \quad \Leftrightarrow \quad v = v_0 + \int_0^t a(t)dt$$

$$v = v_0 + a\int_0^t dt$$

$$\boxed{v = v_0 + at}$$

$$v = \frac{dx}{dt} \quad \Leftrightarrow \quad x = x_0 + \int_0^t v(t)dt$$

$$x = x_0 + \int_0^t (v_0 + at)dt$$

$$\boxed{x = x_0 + v_0 t + \frac{1}{2}at^2}$$

Basic math: $\int x^n dx = \frac{x^{n+1}}{n+1}$

One Dimensional Constant acceleration.

$$\boxed{v = v_0 + at} \qquad \boxed{x = x_0 + v_0 t + \frac{1}{2}at^2}$$

From these we can derive a couple more useful equations:

$$\boxed{\bar{v} = \frac{v + v_0}{2}} \qquad \bar{v} = \frac{x - x_0}{t - 0} = \frac{v_0 t + \frac{1}{2}at^2}{t} = v_0 + \frac{1}{2}at = v_0 + \frac{1}{2}(v - v_0) = \frac{v + v_0}{2}$$

$$\boxed{v^2 - v_0^2 = 2a\Delta x} \qquad t = \frac{v - v_0}{a} \Rightarrow x - x_0 = v_0\left(\frac{v - v_0}{a}\right) + \frac{1}{2}a\left(\frac{v - v_0}{a}\right)^2$$

$$= \frac{vv_0 - v_0^2}{a} + \frac{v^2 + v_0^2 - 2vv_0}{2a}$$

$$= \frac{v^2 - v_0^2}{2a}$$

One Dimensional Constant acceleration.

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v = v_0 + at$$

$$v^2 - v_0^2 = 2a\Delta x$$

$$\bar{v} = \frac{v + v_0}{2}$$

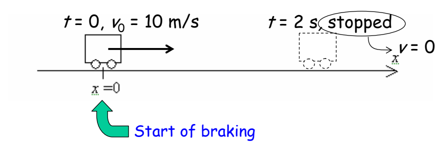
EXAMPLE: Braking Car

A car is traveling with v_0 = 10 m/s. At t = 0, the driver puts on the brakes, which slows the car to a stop in 2 seconds.

a. What is the acceleration produced by the brakes?

- "Translate" the problem –understand it: Draw a figure.

Identify and include initial (t = 0, v_0 = 10 m/s) and final situation (t = 2s; car stopped)



A car is traveling with v_0 = 10 m/s At t = 0, the driver puts on the brakes, which slows the car to a stop in 2 seconds.

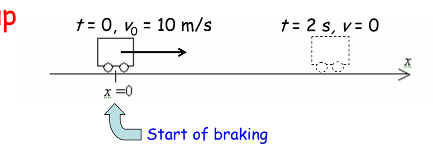
a. What is the acceleration produced by the brakes?

- Identify:

  - We shall **assume that the acceleration is constant**… because otherwise we cannot do the problem! This is just an *approximation* (actually not a very good one for a real car, so the result is a rough approximation to the real thing…). Physics is all about being able to see when we can or cannot do an approximation.

  - **What we are looking for**? The acceleration a

- Setup



Is there an equation that relates my data to the acceleration?

Yes: $v = v_0 + at$

- Execute:

Substituting into the equation:

$$a = \frac{v - v_0}{t} = \frac{0\text{ m/s} - 10\text{ m/s}}{2\text{ s}} = -5\text{ m/s}^2$$

We're not done!!!!

- Evaluate and check:
  - The acceleration is -5 m/s² in the coordinate system we are using.
  - Does the result make sense?
    - Units (ok, m/s²)
    - Sign (ok, it is slowing down).
    - Sanity check on magnitude of acceleration (we'll learn in a few minutes that the acceleration of gravity is ~ 10 m/s²)

A car is traveling with v_0 = 10 m/s At t = 0, the driver puts on the brakes, which slows the car to a stop in 2 seconds.

b. How far does it travel before it stops?

- Identify: We want to find the position when v = 0.
- Setup: Equation that relates the data to the final position: $\Delta x = \frac{v^2 - v_0^2}{2a}$
- Execute: Using this relation

$$\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{0 - (10 m/s)^2}{-2 \times 5 m/s^2} = 10 \text{ m}$$

- Evaluate: The car travels 10 m between the start of braking and the final resting place of the car.
  Check: Units, sign, magnitude…

ACT: Ball going up and down

When throwing a ball straight up, which of the following is true about its velocity v and its acceleration a at the highest point of its path?

A. Both $v = 0$ and $a = 0$

B. $v \neq 0$ but $a = 0$

C. $v = 0$ but $a \neq 0$

Going from $v > 0$ to $v < 0$ (or the other way around)

Velocity is changing

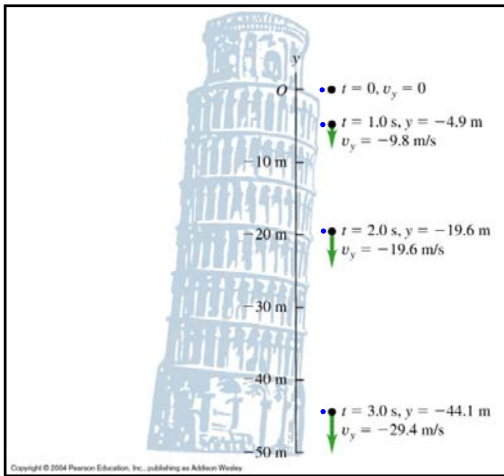
Free fall.

When an object is released in the air, it falls down with a constant acceleration $a = g$ = 9.81 m/s² (as observed by Galileo(1564-1642))

Any two objects, <u>regardless of the mass or composition of the objects</u>, released from a given height will take equally long to reach the floor:

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

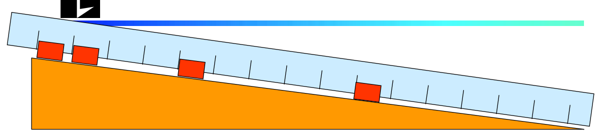
(same acceleration, same initial and final positions, same initial velocity for both, so same t for both!). Is this true?



DEMO:
Coin and cotton ball falling in vacuum

VIDEO:
On the Moon

DEMO: Slanted track



t (s)	x (cm)	$a = 2x/t^2$
0	0	-
5		
10		
15		

1D motion with constant acceleration a

With $x = 0$, $v_0 = 0$, $x = \frac{1}{2}at^2$

ACT: Throwing an Object Downwards

If you drop an object in the absence of air resistance, it accelerates downward at 9.8 m/s². If instead you throw it downward, its downward acceleration after release is:

1. less than 9.8 m/s².
2. 9.8 m/s².
3. more than 9.8 m/s².

This affects the initial **velocity**.

The acceleration in free fall is **always** 9.8 m/s² pointing down.

Example: Free fall

A ball is thrown vertically up into the air (hard!) and comes back down to its starting position 14 s later. At the highest point of its trajectory, how high was the ball?

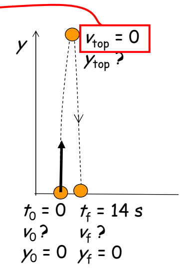
A. 3.3 m

B. 9.8 m

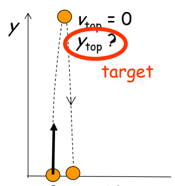
C. 120 m

D. 240 m

E. 480 m



A ball is thrown vertically up into the air (hard!) and comes back down to its starting position 14 s later. At the highest point of its trajectory, how high above the starting position was the ball?



$t_0 = 0$ $t_f = 14$ s
v_0 ? v_f ?
$y_0 = 0$ $y_f = 0$

To find y_{top}, I can use: $v^2 - v_0^2 = -2g\Delta y$ (?)

[Use $y = y_0 + v_0 t - \frac{1}{2}gt^2$ for the final situation:

$$0 = 0 + v_0 t_f - \frac{1}{2}gt_f^2$$

$$v_0 = \frac{gt_f}{2} = \frac{(9.8\ \text{m/s}^2)(14\ \text{s})}{2} = 68.6\ \text{m/s}$$]

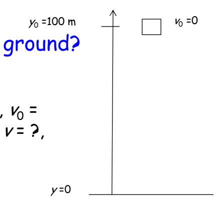
Then, $0^2 - v_0^2 = -2g\left(y_{top} - 0\right)$

$$y_{top} = \frac{v_0^2}{2g} = \frac{(68.6\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = \boxed{240\ \text{m}}$$

(Answer D)

EXAMPLE: King Kong drops a car

King Kong drops a car from 100 m.
a. How long does it take to hit the ground?



Draw a figure, select axis.

Identify initial situation ($t = 0$, $y_0 = 100$ m, $v_0 = 0$) and final situation (hit the ground; t =? v = ?, $y = 0$)

What are we looking for? t

Find an equation that relates data to t:

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

$$0 = 100\text{m} + 0 + \frac{1}{2}(-9.8\ \text{m/s}^2)t^2$$

$\boxed{t = 4.5\ \text{s}}$

Checks: Units are ok, it's positive, it's reasonable…

King Kong drops a car from 100 m.

b. What is the velocity of the car right before it hits the ground?

$y = 0$

We want an equation with v, a and y: $v^2 - v_0^2 = 2a\Delta y$

$$v^2 - v_0^2 = 2a\Delta y$$

$$v = \pm\sqrt{2a\Delta y + v_0^2} = \pm\sqrt{2(-9.8\ \text{m/s}^2)(0 - 100\ \text{m})} = \pm 44.3\ \text{m/s}$$

We can't have two answers, we have to choose a sign. Look at your figure: the positive axis points up. So the answer is –44.3 m/s.

ACT: Change in v^2

The graph below shows a plot of acceleration versus position for an object. When the object passes $x = 0$, its velocity is 2 m/s pointing in the $+x$ direction. What is the velocity squared when the object arrives at $x = 10$ m?

1. v^2 = 8 m²/s²
2. v^2 =12 m²/s²
3. v^2 = 20 m²/s²
4. v^2 = 44 m²/s²
5. v^2 cannot be determined.