Lecture 3
1D motion

ACT: Cross product

Vectors A, B and C are on the plane of the screen. They are drawn to scale. Compare the magnitude of these two cross products:

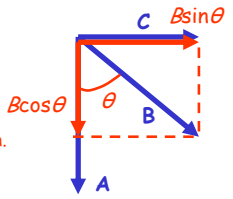
A. $|A\times B| > |A\times C|$

B. $|A\times B| = |A\times C|$

C. $|A\times B| < |A\times C|$

$|A\times B| = AB\sin\theta = AC = |A\times C|$

And they both point out of the screen.



The cross product selects the part of ***B*** that is perpendicular to the direction of ***A***.

An overview of Mechanics

Mechanics: study of the motion of objects.

1. **Kinematics**: How do things move?
2. **Dynamics**: Why do things move?
3. **Conservation laws**:
   - Work-energy
   - Momentum (linear and angular)
4. **Some special cases**:
   - Rigid body motion
   - Simple harmonic motion
   - Gravitational forces, Hooke's law…

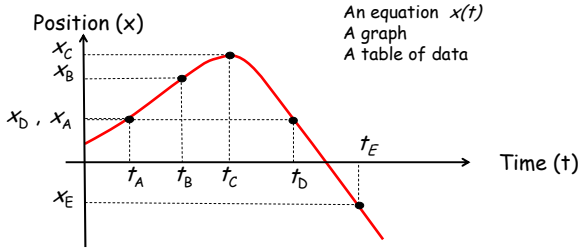
For some problems, the math is easier to handle with

The Simplest Case:
1D Motion along a Straight Line

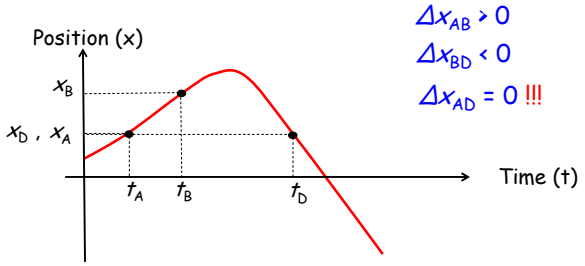
Position: A single + or – number (coordinate) describes the location of an object relative to a reference point (origin).

Description of motion: position *x* as a function of time *t*:

An equation *x(t)*
A graph
A table of data



Displacement: Change in position $\Delta x_{AB} = x_B - x_A$



$\Delta x_{AB} > 0$

$\Delta x_{BD} < 0$

$\Delta x_{AD} = 0$!!!

Displacement only tells us about the "result" of the motion (not the path).

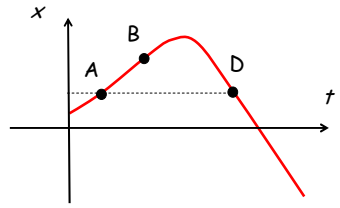
Velocity

= rate of change of position

Simplest idea:
For displacement of Δx over a time Δt:

$$\bar{v} = \frac{\Delta x}{\Delta t}$$ **Average velocity**

Unit: m/s



$\bar{v} = \frac{\Delta x}{\Delta t}$

For $\Delta t = t_B - t_A \rightarrow \bar{v} > 0$

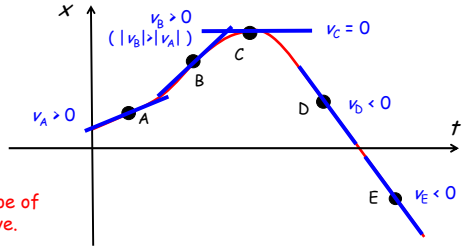
For $\Delta t = t_D - t_B \rightarrow \bar{v} < 0$

For $\Delta t = t_D - t_A \rightarrow \bar{v} = 0$!!!

Not a very good description of what really happened!
Problem: Our time intervals are too large and too many things are happening in between.

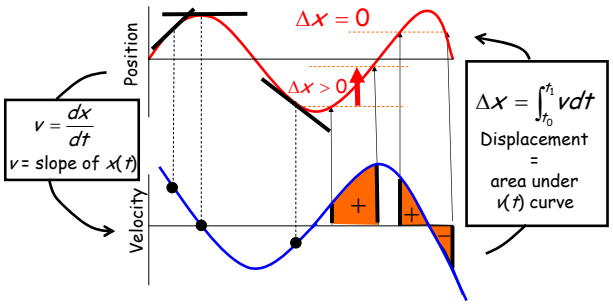
Instantaneous Velocity

Solution: Take limit $\Delta t \rightarrow 0$. $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$



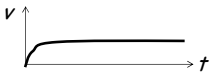
It is the slope of the *x(t)* curve.

$x \leftrightarrow v$

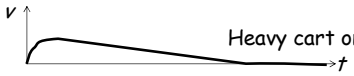


Examples: $v(t)$ graphs

Glider on air track

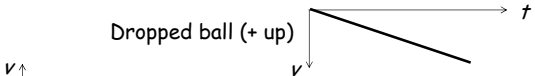


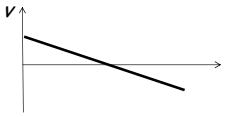
DEMO



Heavy cart on floor

Dropped ball (+ up)





Ball going up and down (+ up)

Velocity versus speed

Velocity = vector (or value including sign, in 1D)

Speed = magnitude of velocity (always positive)

Example: 1D motion

An object moves along the x-axis as follows:

$x(t) = 25 + (10\ \text{m/s})t - (30\text{m/s}^3)t^3$

a) At what time ($t > 0$) does the object stop?

Velocity = 0

a. 0 s
b. 0.33 s
c. 3.3 s
d. 10 s
e. 33 s

$$v(t) = \frac{dx(t)}{dt} = 10\ \text{m/s} - (90\ \text{m/s}^3)t^2$$

$$v = 0 \quad \Rightarrow \quad 10 - 90t^2 = 0$$

$$t = \sqrt{\frac{10}{90}} = 0.33\ \text{s}$$

An object moves along the x-axis : $x(t) = 25 + (10\ \text{m/s})t - (30\text{m/s}^3)t^3$

b) What is the average velocity in this time interval?

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{(27.2 - 25)\ \text{m}}{0.33\ \text{s}} = 6.7\ \text{m/s}$$

$x(t = 0) = 25\ \text{m}$

$x(t = 0.33\ \text{s}) = 27.2\ \text{m}$

Note! $\bar{v} \neq \frac{v(0.33\ \text{s}) + v(0)}{2}$ $\left(= \frac{0 + 10\ \text{m/s}}{2} = 5.0\ \text{m/s}\right)$

Acceleration

= rate of change in velocity

Average acceleration: $\bar{a} = \frac{\Delta v}{\Delta t}$ Unit: m/s²

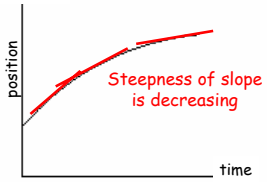
Instantaneous acceleration: $a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

The instantaneous acceleration is the slope in $v(t)$ curve.

The change in velocity Δv is the area under the $a(t)$ curve.

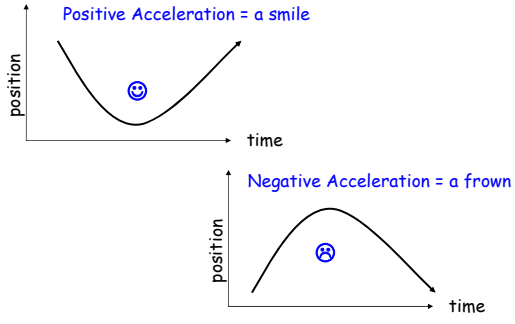
ACT: Tracking a Train

A train car moves along a long straight track. The graph shows the position as a function of time for this train. The graph shows that the train:



1. speeds up all the time.
2. slows down all the time.
3. speeds up part of the time and slows down part of the time.

Positive and Negative Acceleration



Positive vs Negative

The car is moving forwards and you step on the gas

$x > 0$
$v > 0$
$a > 0$

Positive vs Negative

The car is moving forwards and you step on the gas

$x < 0$
$v > 0$
$a > 0$

Positive vs Negative

The car is moving forwards and you step on the brakes

$x > 0$
$v > 0$
$a < 0$

ACT: Positive vs Negative

The car is moving backwards and you step on the gas. Which of the following is correct?

A. $v < 0, a > 0$
B. $v < 0, a < 0$
C. $v > 0, a < 0$

$x > 0$
$v < 0$
$a < 0$

Positive vs Negative

For 1D motion along a straight line,

Speeding up: Velocity and acceleration have same sign.

Slowing down: Velocity and acceleration have opposite sign.